Mathematical Tables *and other* Aids to Computation

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and Other Aids to Computation
by

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Mathematical Tables on Punched Cards

The increasing number of scientific computing laboratories employing the punched card method in general scientific calculation indicates the need for more general dissemination of information concerning mathematical tables on punched cards. In the past when the number of such laboratories was small, the exchange of information was made by personal contact and informal correspondence. It is felt that a wider distribution of this information will not only facilitate the exchange of card files between laboratories but may, on occasion, make it possible for a scientist without punched card facilities to utilize a printed list of selected data from the cards at one of the laboratories.

One of the functions of the new Watson Scientific Computing Laboratory at Columbia University will be to maintain a card file of current information concerning mathematical tables on punched cards at the various laboratories. This information will be available by correspondence, and such portions as are suitable may be published from time to time in *MTAC*.[1]

The question of how much of this detailed material should be published is difficult and can probably be answered only by experience; the comments of the readers will, no doubt, influence future publication. Owing to the ease of constructing and re-arranging tabular data on punched cards, the tables tend to be more fluid in nature than printed tables and to reflect not only the problem for which they were constructed but the particular equipment which was available. These and other factors affect the problem of maintaining a current detailed printed description of such tables.

The following list of tables has been prepared from material on file in World Headquarters of the International Business Machines Corporation. The list is not complete, but it indicates some of the laboratories and the types of tables in use. Many laboratories are, of course, engaged in secret war work and it would be impossible now to give a complete list of the tables in use or even of the laboratories themselves.

ABERDEEN PROVING GROUND, BALLISTIC RESEARCH LABORATORY

File	Function and Accuracy	Range and Interval
1	$\tan^{-1} x$ in degrees (2D)	0(.001)3.75(.01)18.25
2	$\sin x$ (5D), $\cos x$ (5D), $\tan x$ (5D)	0(0°.01)45°
3	$\int_0^x \sqrt{1+x^2}\,dx$ (6D)	0(.1)20
[2]4	$\sqrt{x}$ (8D)	.0001(.0001)1(.001)10
5	x^2, x^3, x^4, x^5	0(.0001)1
[2]6	$1/x$ (6D)	.1(.0001)1
7	$1/x$ (8D)	.001(.001)1
8	$\sqrt[3]{x}$ (7D)	1(.001)10
[2]9	$\log x$ (7D)	1 (.0001)10
10	Bessel functions $e^{-x}I_0(x)$, $e^{-x}I_1(x)$, $e^{-x}K_0(x)$, $e^{-x}K_1(x)$ and e^x (all to 7D)	0(.02)10

[1] For discussion connected with punched-card tables, see *MTAC*, p. 173f, 334.

[2] These tables are provided with second order interpolation coefficients $A = \frac{1}{4}(\Delta_{i-1}+\Delta_i)$, $B = \frac{1}{4}\Delta^2_{i-1}$.

File	Function and Accuracy	Range and Interval
11	Numerous extensive ballistic tables	
12	Lagrangean interpolation coefficients	
	3 point (5D)	0(.001)1
	5 point (7D)	0(.001)2

See also *MTAC*, p. 332.

ARTHUR D. LITTLE, INC., CAMBRIDGE, MASS.

File	Function and Accuracy	Range and Interval	Remarks
1	$x, x - 273, x^2, x^3, x^4, 1/x, \log x, \ln x$; $kx, 1/(kx)$ All 8S	$x = .1(1)1500$ $k = .6951765, .8617013, 1.380474, 1.986467$	For use in thermodynamics

CALIFORNIA INSTITUTE OF TECHNOLOGY, GATES & CRELLIN LABORATORIES OF CHEMISTRY

File	Function and Accuracy	Range and Interval
1	$A \sin 2\pi hx, A \cos 2\pi hx$	$x = 0(.002).25$ $h = 0(1)30$ $A = \pm 1(1)5; 10(10)50; 100, 200; 500.$
	Accuracy (.01) except for $A = \pm 100(.005)$ and $\pm 500(.025)$	
2	$A \sin 2\pi hx$	$x = 0(.05)7.15$ $h = 0(.01)5$ $A =$ same as 1 with addition of $\pm 300, \pm 400$.

COLUMBIA UNIVERSITY, THE THOMAS J. WATSON ASTRONOMICAL COMPUTING BUREAU

File	Function and Accuracy	Range and Interval	Source
1	$\sin x$ (7D), $\cos x$ (7D), $\tan x$ (7D)	$0(0°.01)90°$	Peters
2	$\tan x$ (7D)	$0(10'')30°$	Brandenburg
3	$\tan x$ (7D), $\sec x$ (7D)	$0(1^s)20^m$	
4	$\tan x$ (8D), $\sec x$ (8D)	$0(0^s.1)20^m$	
5	$x^{-3/2}$ (8D)	2.0(.001)7.5 7.5(.01)20	Comrie
6	$1/n$ (10D), n^2	1000(1)9999	
7	Numerous extensive astronomical tables.		

INTERNATIONAL BUSINESS MACHINES CORPORATION, NEW YORK SERVICE BUREAU

File	Function and Accuracy	Range and Interval
1	$1/x$ (7S, or more)	1(1)10000

MASSACHUSETTS INSTITUTE OF TECHNOLOGY, DEPARTMENT OF ELECTRICAL ENGINEERING

File	Function and Accuracy	Range and Interval	Source
1	$\sin nx$ (7D): $n = 1, 2, \cdots 9$	$0(0°.1)360°$	Peters
2	$\cos x$ (7D)	$0(0°.1)90°$	Peters
3	$\sin x$ (8D), $\cos x$ (8D)	0(.1)20 rad.	NYMTP
4	$\sin x/x$ (8D), $\cos x/x$ (8D)	0(.1)20 rad.	M.I.T.
5	$d^n/dx^n (\sin x/x)$ (8D): $n = 1, 2, \cdots 8$	0(.1)20 rad.	M.I.T.
6	$1/x$ (8D)	1(1)2500	Barlow
7	$1/x$ (8D)	1(.1)20	Barlow
8	$J_0(x)$ (10D), $J_1(x)$ (10D), $Y_0(x)$ (8D)	0(.01)25	B. A. Tables VI

NATIONAL DEFENSE RESEARCH COMMITTEE, APPLIED MATHEMATICS PANEL

File	Function and Accuracy	Range and Interval
1	$\log x$ (7D) (5D)	100(1)9999

PRINCETON UNIVERSITY, NATIONAL DEFENSE RESEARCH COMMITTEE

File	Function and Accuracy	Range and Interval
1	$\sqrt{x}$ (9D) (6D), $\sqrt{10x}$ (8D) (6D), $1/x$ (7D) (6D), $\Delta\sqrt{x}$, $\Delta\sqrt{10x}$, $\Delta(1/x)$	1(1)9999
2	$\log x$ (5D), $\Delta \log x$, $1/(\Delta \log x)$, $\log x/(\Delta \log x)$	1(1)1000
3	$\log \sin x$ (6D), $\log \cos x$ (6D), $\log \tan x$ (6D), $\log \operatorname{ctn} x$ (6D)	0(1)800 mils
4	$1600 - x$, $\sin x$, $\cos x$, $\tan x$, $\operatorname{ctn} x$, $\sec x$, $\csc x$, $\frac{1}{2}\tan x$, $\frac{1}{2}\operatorname{ctn} x$ (all 5D); $\log \sin x$, $\log \cos x$, $\log \tan x$. See *MTAC*, p. 146	0(.1)1600 mils

UNIVERSITY OF MICHIGAN

File	Function and Accuracy	Range and Interval
1	$x^i y^j : i + j = 2, 3, 4$	$x, y = 0(1)9$
2	$x^2, x^3, x^4, 1/x$ (9D), $\sum_{n=1}^{x} 1/n$ (9D), $\sqrt{x}$ (7D), $\log x$ (7D)	1(1)99
	$\sqrt{1 - x^2}$ (4D), $1 - \sqrt{1 - x^2}$ (4D), $\int_0^x e^{-\frac{1}{2}x^2}dx/\sqrt{2\pi}$ (5D)	.01(.01).99
3	$x^i y^j z^k : i + j + k = 2, 3$	
	$n = 100\,x + 10\,y + z$, n^2, n^3, $1/n$ (9D),	$x, y, z = 0(1)9$
	$\log n$ (4D), $\log^{-1}(n/1000)$ (4D)	0(1)999

U. S. LAKE SURVEY, MILITARY GRID UNIT

File	Function and Accuracy	Range and Interval
1	$\tan x$ (12D), $\sin x$ (12D), $\Delta \sin x$, $\Delta \tan x$	0(10″)20°
2	$\sec x$ (10D), $\csc x$ (10D)	0(1′)90°
3	$\sin x$ (15D), $\cos x$ (15D), $\sin^2 x$ (10D), $\cos^2 x$ (10D), $\sin x \cos^2 x$ (7D)	0(1′)90°
4	$\sin 2x$ (10D) (8D), $\sin 4x$ (7D) (5D), $\sin 6x$ (6D) (4D), $\sin 8x$ (7D)	0(1′)90°
5	$\sin x$ (12D), $\sin 2x$ (10D), $\sin 4x$ (8D), $\sin 6x$ (8D), $\sin 8x$ (8D)	0(1′)90°
6	$\cos x$ (6D) (5D)	0(1′)90°
	$1 - \cos x$ (5D)	90°(1′)180°
7	$\cos x$ (6D) (5D), $1 - \cos x$ (5D)	90°(1′)180°
8	$\cos^{-1} x$ (5D) (critical)	0(1′)180°
9	$\tan x$ (15D) (8D), $\cot x$ (15D) (8D)	0(1′)90°
	$\tan^2 x$ (6D)	0(1′)88°
10	$\cos x$ (15D), $\Delta \cos x$ (15D)	0(1′)90°
11	$\sin x$ (9D) (8D) (7D) (6D), $\Delta \sin x$ (7D), $\Delta \sin x$ for 1″ (10D) (8D) (9D)	0(1′)90°
12	$x!$	1(1)100
	$x^2, x^3, x^4, 1/\sqrt{x}$ (7D), $\sqrt{x}$ (7D), $\sqrt[3]{x}$ (7D), $1/x$ (9D)	1(1)999
	$x^2, x^3, \sqrt{x}$ (6D), $\Delta\sqrt{x}$, $\sqrt{10\,x}$ (6D), $\Delta\sqrt{10x}$, $\sqrt[3]{x}$ (6D), $\Delta\sqrt[3]{x}$, $1/x$ (10D), $\Delta(1/x)$	1000(1)5700

Spheroid functions (notation as in U.S.C. & G.S.S.P. no. 8) for the following spheroids, Clarke 1880, Everest, Bessel, Hayford, Helmert, Airy, Clarke 1858, Plessis, Danish, Struve:

File	Function and Accuracy	Range and Interval
13	meridional arc (.001 meters)	0(1′)90°
	A (10D, 8S), B (10D, 8S)	0(1′)90°
14	C (15D, 7S), D (12D, 4S), E (17D, 5S), F (16D, 4S)	0(1′)90°
15	N, R (both .001 meters)	0(1′)90°
16	military grid coordinates from U.S.C. & G.S.S.P. no. 59	
	X, Y (both .1 yard) Lat., Long. of U. S.	5′ intersections
17	latitude transformation tables geometric-isometric: isometric-geodetic for following spheroids: Clarke 1866, 1880, 1860; Hayford, Bessel, Everest	0(1′)90°
	geodetic-authalic: authalic-geodetic for Clarke 1866	0(1′)90°

U. S. NAVAL OBSERVATORY

File	Function and Accuracy	Range and Interval
1	$\sin x$ (7D), $\cos x$ (7D), $\tan x$ (7D)	0(0°.01)90°
2	$1/x$ (6D)	1(.001)4
		4(.01)10
3	$\log^{-1} x$ (8D)	0(.001)1
4	$\text{hav}^{-1} x$ (about 0°.001)	0 to 25°
5	$\sin x$ (8D), $\cos x$ (8D)	0(0°.01)90°
	Special tables for astronomical work.	

VEGA AIRCRAFT CORPORATION

File	Function and Accuracy	Range and Interval
1	$\log x$ (8D), $\Delta \log x$ (8D), $1/(\Delta \log x)$	10000(1)100000

W. J. E.

RECENT MATHEMATICAL TABLES

211[A].—W. Vriesendorp, *The Calculating Dictionary*, Utrecht, L. E. Bosch & Zoon, printer, for the author, 1937. [1003 p.], 13.5 × 26.7 cm. 17.5 Dutch florins.

This *Dictionary* contains the multiplications up to 999 × 999, except multiples of ten. There are two pages of examples showing applications of the volume in obtaining the products of larger numbers. On each page headed 2, 3, . . ., 999 are given 900 products. The Dutch edition has the title *De Rekendictionnaire Tabellen bijeengebracht ten dienste van handel, industrie, administratie en onderwijs*.

There is no reference to any other work offering the same tabular information, and readily available in 1937, such as: A. L. Crelle, *Calculating Tables*, Berlin, 1930 which has gone through many editions, since the first two-volume edition in 1820, including one by an insurance company in Japan (1913).

212[D].—M. J. Buerger & Gilbert E. Klein, "Correction of X-ray intensities for Lorentz and polarization factors," *Journal of Applied Physics*, v. 16, July, 1945, p. 414–418. 20 × 26.6 cm.

Four-place tables are given for the following functions: $2 \sin 2\theta/(1 + \cos^2 2\theta)$ and $2/(1 + \cos^2 2\theta)$, for the argument $\sin \theta = 0(.001).999$.

213[D, M].—LEON BESKIN, "General circular ring analysis," *Aircraft Engineering*, v. 17, May, 1945, p. 127–132. 24.5 × 31 cm.

There are tables of $\phi(\theta) = [1/2\pi][\theta(1 + \cos\theta) - \frac{1}{2}(3\sin\theta)]$, $\phi'(\theta) = [1/2\pi][-\theta\sin\theta - \frac{1}{2}\cos\theta + 1]$, $\phi''(\theta) = [1/2\pi][-\frac{1}{2}\sin\theta - \theta\cos\theta]$, $\phi'''(\theta) = [1/2\pi][-\frac{1}{2}(3\cos\theta) + \theta\sin\theta]$, $\psi(\theta) = \phi(\theta) + \phi''(\theta) = [1/2\pi][\theta - 2\sin\theta]$, $\psi'(\theta) = \phi'(\theta) + \phi'''(\theta) = [1/2\pi][1 - 2\cos\theta]$, $\psi''(\theta) = \phi''(\theta) + \phi''''(\theta) = [1/\pi]\sin\theta$, to 5D, $\theta = 0(5°)180°$. Graphs of these seven functions are also set forth. Furthermore, there are tables and graphs of the following nine functions:

$$\Gamma(a) = [1/2\pi][\cos a(\pi^2/6 - 23/8 - \tfrac{1}{2}\pi a + a^2/4) + \tfrac{1}{2}(\pi - a)3\sin a + \pi^2/3 - 2 - a\pi + \tfrac{1}{2}a^2],$$
$$\Gamma_1(a) = [1/2\pi][(\pi - a)(\cos a - 1) - \sin a(\pi^2/6 - 11/8 - \tfrac{1}{2}\pi a + a^2/4)],$$
$$\Gamma_2(a) = [1/2\pi][-\cos a(\pi^2/6 - 3/8 - \tfrac{1}{2}\pi a + a^2/4) - \tfrac{1}{2}(\pi - a)\sin a + 1],$$
$$\Gamma_3(a) = [1/2\pi]\sin a(\pi^2/6 + 1/8 - \tfrac{1}{2}\pi a + a^2/4),$$
$$\Gamma_4(a) = [1/2\pi][\cos a(\pi^2/6 + 1/8 - \tfrac{1}{2}\pi a + a^2/4) - \tfrac{1}{2}(\pi - a)\sin a],$$
$$\Lambda(a) = [1/2\pi][\cos a(\pi^2/6 + 13/8 - \tfrac{1}{2}\pi a + a^2/4) + \tfrac{1}{2}(\pi - a)\sin a],$$
$$\Delta(a) = [1/2\pi][-\tfrac{1}{2}(5\cos a) + (\pi - a)\sin a + \pi^2/3 - 1 - a\pi + \tfrac{1}{2}a^2],$$
$$E(a) = [1/2\pi][-2\cos a + \pi^2/3 - a\pi + \tfrac{1}{2}a^2],$$
$$\Omega(a) = [1/\pi]\cos a,\ a = 0(5°)180°, \text{ all to 5D, except } \Gamma(a) \text{ and } \Gamma_1(a), \text{ to 6D.}$$
$$\Gamma(a) = \int_0^{2\pi}\phi(\theta)\phi(\theta + a)d\theta,\quad \Gamma_4(a) = \int_0^{2\pi}\phi''(\theta)\phi''(\theta + a)d\theta,$$
$$\Delta(a) = \int_0^{2\pi}\phi(\theta)\psi(\theta + a)d\theta, \text{ etc.}$$

214[E].—H. W. HOLTAPPEL, *Tafels van* e^x. Groningen, Noordhoff, 1938, xxxi, 132 p. 24 × 29.5 cm. 6.00 Dutch florins.

The book under review provides a very useful table of the exponential function e^x, both for positive and negative values of the argument, over ranges which hitherto had been accessible only with difficulty in existing tables. In the preface, which is published in Dutch, English, French, German, and Italian, the author explains the origin of the present volume.

Intrigued by the apparent scarcity of tables of the exponential function the author made a cursory survey of the literature and discovered the existence of the extensive computations of F. W. NEWMAN: "Table of the descending exponential function to twelve or fourteen places of decimals," Cambridge Phil. So., *Trans.*, v. 13, 1883, p. 145–241. Unfortunately he failed to find Newman's second paper: "Table of the exponential function e^x to twelve places of decimals," Cambridge Phil. So., *Trans.*, v. 14, 1889, p. 237–249; see N43 in this issue. Hence he says: "Seeing that the *Encyclopædia Britannica*, 1929 mentioned Mr. Newman's tables as the most extensive published concerning e^x, I was convinced that a table as desired was not in existence. I therefore resolved to compile one myself."

The author incorporated the values of e^{-x} from Newman's table, which he checked and found accurate except for an occasional value in the 12th decimal place. His principal table, namely values of the exponential over the range $x = 0(.001)9.999$, he constructed from 13-place values over the range $x = 0(.01)9.99$ by means of the formula

$$e^{x+h} = e^x\left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \cdots\right),$$

and in this way "built up the whole as one linked chain."

His check of Newman's table and of his own computation was effected through the formula

$$(e^n - 1)/(e - 1) = e^{n-1} + e^{n-2} + \cdots.$$

With this introduction explaining the origin of the tables we may turn to their actual contents.

Table I, which is the author's principal table, gives in adjoining columns the values of e^x and e^{-x} to 10D for $x = 0(.001)9.999$.

Table IIA gives values of e^x for $x = 1(1)24$ to 20D and e^{-x} over the same range from

16 to 22D. Tables IIB, C, and D tabulate the same functions over the ranges $x = .1(.1).9$; $.01(.01).09$; $.001(.001).009$, the first function to 24 or 25D and the second to 18D.

Tables IIIA, B, C give values of e^x over the following ranges: $x = [0(.1)9.9; 18D]$, $x = [0(.001).099; 24D]$, $x = [0(.001).999, 17D]$.

Tables IIID, E, and F tabulate e^y, where y is successively $x \cdot 10^{-6}$, $x \cdot 10^{-9}$, and $x \cdot 10^{-12}$ over the range $x = [0(1)999; 20D]$.

Table IV provides to 18D the first ten multiples of M and $1/M$, where $M = \log e$.

The errors of this volume are discussed elsewhere in this issue, MTE 68.

In the year following the publication of the work under review, the extensive tables of the NYMTP made their appearance. These cover the following range: For e^x, $x = [0(.0001)1; 18D]$, $x = [1(.0001)2.5(.001)5; 15D]$, and $x = [5(.01)10; 12D]$; for e^{-x}, $x = [0(.0001)2.5; 18D]$. It is thus seen that, to 10 decimal places, the Holtappel tables, even now, extend the range of values tabulated, especially for the exponent of descending argument.

H. T. D.

215[E].—NYMTP (A. N. LOWAN, technical director), *Tables of the Exponential Function e^x*, New York, 1939, xviii, 535 p. 21 × 27 cm. Reproduced by a photo-offset process. Sold by the U. S. Bureau of Standards, Washington, D. C. $2.00.

This volume provides accurate tables of the function e^x for small intervals of the argument. The frequent use of the exponential function in a wide variety of computational work makes these tables extremely valuable as they give a complete tabulation of the function over a large range of values. An account of the care with which the tables were designed and checked is set forth in the Introduction.

The first four tables give the values of the ascending exponential from 0 to 10 with varying intervals and number of decimals. The ranges are as follows: $x = [0(.0001)1; 18D]$, $[1(.0001)2.5(.001)5; 15D]$, and $[5(.01)10; 12D]$.

The values of the descending exponential are given for the range $-x = [0(.0001)2.5; 18D]$.

Besides these main tables, there are three tables to cover more adequately the interval about zero, and certain integral values of x, namely: $\pm x = [0(.000001).0001; 18D]$, $[1(1)100; 19S]$, and $[1(1)9 \cdot 10^{-p}; 18D]$, $p = 7(1)10$.

It is unfortunate that the tables for e^{-x} were not extended beyond $x = 2.5$. The values of the function for arguments larger than 2.5 can, of course, be calculated by using the supplementary integer table. To be sure there are Newman's tables[1] for $x = [0(.001)15.349; 12D]$, $[15.350(.002)17.3(.005)27.635; 14D]$, and $[.1(.1)37; 18D]$. However, it would have been a decided advantage to have had the complete tables collected into one volume.

EVELYN FIX

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[1] F. W. NEWMAN, "Table of the descending exponential function to twelve or fourteen places of decimals," Cambridge Phil. So., *Trans.*, v. 13, 1883, p. 145–241.

216[E, L].—NATIONAL DEFENSE RESEARCH COMMITTEE, *Tables for Solutions of the Wave Equation for Rectangular and Circular Boundaries having Finite Impedance*, prepared by A. N. LOWAN for the NYMTP, & P. M. MORSE, H. FESHBACH & E. HAURWITZ for the M.I.T. Underwater Sound Laboratory. Report dated June 1945. Printed from manuscript, ii, 52 leaves, and 7 plates on only one side of each leaf, by the photo-offset process. These tables are available only to certain Government agencies and activities.

For rectangular boundaries the wave equation can be separated into differential equa-

tions of the form

(1) $$d^2\psi/dx^2 + k^2\psi = 0.$$

The boundaries for the x coordinate can be placed at $x = 0$, and $x = a$. The boundary condition at $x = a$ will be defined by the equation

(2) $$\psi = zx(d\psi/dx), \quad (x = a),$$

where $z = re^{i\phi}$ is proportional to the impedance of the boundary at $x = a$. The solution of the problem depends on the nature of the boundary conditions at $x = 0$.

Case I. Boundary condition at $x = 0$ is $\psi = 0$.

In this case the solution used is

(3) $$\psi = \sinh (ikx) = \sinh (wx/a),$$

where $w = ika$. The boundary condition at $x = a$, given by the equation (2), then gives rise to the equation

(4) $$(1/w) \tanh w = z = re^{i\phi},$$

where

(5) $$w = R - iI = \pi\beta e^{-i\phi}.$$

The values of R and I, for each of four branches are given, Table I, p. 7–26, as functions of r and ϕ, $r = [0(.01).2(.02).5(.05)1(.1)2(.2)5(.5)10; 3D]$, $\phi = 0(10°)180°$. After p. 48 is a large folding plate (18 × 67 cm.) containing a graph of (4). There are also two other plates of this function, $w = \pi\beta e^{-i\phi}$, First Branch; and, $z = re^{i\phi}$, Second Branch.

Case II. Boundary condition at $x = 0$ is $d\psi/dx = 0$.

In this case the solution used is

(6) $$\psi = \cosh (ikx) = \cosh (wx/a), \quad w = ika.$$

This gives rise to the equation

(7) $$(1/w) \coth w = z = re^{i\phi}.$$

Solutions of this equation for

$$w = R - iI = \pi\beta e^{-i\phi},$$

as a function of r and ϕ, are given, as above, in plots and Table II, p. 28–47, and after p. 51.

Both of these transformations have branch points in the complex plane; their position can be determined by setting $dz/dw = 0$. For Case I, this corresponds to setting $2w = \sinh (2w)$, and for Case II it corresponds to setting $2w = -\sinh (2w)$. The positions of the first five branch points for each case are given on p. 4.

The wave equation determining the propagation of circularly symmetrical waves down cylindrical ducts, with a circular cross section, is (the corresponding equation of the publication is in unintelligible form)

(8) $$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\psi}{d\rho}\right) + k^2\psi = 0.$$

The solution of this equation is

$$\psi = J_0(iw\rho/a),$$

$w = ika = R - iI$, a being the radius of the circular cross section.

Taking account of the boundary condition (2) in this case, we are led to the equation

(9) $$iJ_0(iw)/[wJ_1(iw)] = z = re^{i\phi}.$$

The branch points of this transformation are determined by setting $dz/dw = 0$, which

yields

$$J_1(iw) = \pm iJ_0(iw).$$

Three solutions of this equation, determined by interpolation, are given (p. 5). There is also a chart of the transformation (9), where, as in the other charts, 4 heavy dotted lines divide different branches of the transformation.

In Tables I and II are various subtables of branch cuts and branch points. In the neighborhood of the branch points the accuracy of these tables is not better than 1%.

An errata list, recently distributed by P. M. Morse, indicates a number of errors in the tables.

R. C. A.

217[F].—*Polnoe Sobranie Sochineniĭ P. L. Chebysheva* [Complete Collection of Works by P. L. CHEBYSHEV], Volume I: *Teoriía Chisel* [Theory of Numbers]. Moscow and Leningrad, Academy of Sciences, 1944, 342 p. + portrait frontispiece. 17 × 26 cm. 20 roubles paper; 23 roubles bound. Edition of 5000 copies.

The Works (*Sochineniía-Oeuvres*) of PAFNUTIĬ L'VOVICH CHEBYSHEV (1821–1894) were edited by A. A. MARKOV & N. I. SONIN, and published by the Academy of Sciences at St. Petersburg, 1899–1907, in two editions, one entirely in Russian, 2v., the other entirely in French, 2v., large handsome volumes of over 700 p. each. These two editions were made possible when a brother of Chebyshev gave 5000 roubles towards the cost of publication, coupled with the condition as to the two editions. The translations into Russian of Chebyshev's papers originally written in French, and into French of papers originally written in Russian were mainly carried out by Chebyshev's pupils. These volumes did not include Chebyshev's doctoral dissertation on the Theory of Congruences (1849), or his master's thesis Essay of Elementary Analysis on the Theory of Probabilities (1845), or his Lectures on the Theory of Probability (1879–80), published in 1936. Since the original *Sochineniía-Oeuvres* are now difficult to obtain, the Academy of Sciences decided to celebrate the fiftieth anniversary of Chebyshev's death by publishing a more adequate Russian edition of his works, in 5 volumes to include his dissertations and unpublished papers, as well as a complete list of photographs of mechanisms constructed by him. Volume I, containing his work on the Theory of Numbers, and edited by I. M. Vinogradov & B. N. Delone, is the one before us for review; v. II–III are to contain Chebyshev's work on Analysis; v. IV, that on mechanisms; and v. V, other works, and historical and biographical material. The new edition is also to contain short commentaries indicating subsequent development of various themes of Chebyshev in contemporary science.

The editorial committee appointed by the president of the Academy was as follows: S. N. BERNSTEIN (Chairman), N. G. BRUEVICH, I. M. VINOGRADOV, A. N. KRILOV, L. S. LEĬBENZON, S. L. SOBOLEV, N. N. ARMOBOLEVSKIĬ, B. N. DELONE, and V. L. GONCHAROV.

Now for the contents of v. I. The frontispiece and biographical sketch by Posse (p. 5–9), are the same as in v. 2 of the *Sochineniía*. Then comes the sixth edition of the Theory of Congruences,[1] p. 10–172, with Tables, p. 311–339. These four Tables are as follows:

(i) Prime numbers less than 6000;
(ii) Primitive roots and indices of all prime numbers modulo p < 200; these tables were taken from M. OSTROGRADSKIĬ (1838), but were given much more extensively by JACOBI, in his *Canon Arithmeticus*, 1839, p < 1000;
(iii–iv) Linear divisors of the quadratic form $x^2 \pm ay^2$ for all values of a from 1 to 101.

In his *Guide to Tables in the Theory of Numbers*, Washington, D. C., 1941, D. H. L. lists (p. 132–133) numerous errors in (iii–iv) of editions (a) and (f), but states that edition (c) is without error. It is therefore with some surprise that we note the following earlier errors are also in (d), the present edition:

Form	Insert	Delete
$x^2 + 42y^2$	157	159
$x^2 + 66y^2$	71	77
$x^2 + 86y^2$	87	89
$x^2 + 89y^2$	345	354
$x^2 + 101y^2$	309, 317, 333	305, 321
$x^2 - 38y^2$	21, 131	23, 129
$x^2 - 62y^2$	107, 141	103, 145

There is also an error in (ii), which was correct in edition (c), namely:

$p = 13$, T. I, argument 12, *for* –, *read* 6.

Then in the rest of the volume under review are 8 papers (p. 173–282), a demonstration of a theorem of Chebyshev based on a fragment found among his papers, edited by Markov (p. 283–284), and Commentary by Delone (p. 285–309). The paper on Quadratic Forms (p. 208–228) contains a table (p. 220–222) of quadratic forms $\pm (x^2 - 2y^2), \cdots, \pm (x^2 - 33y^2)$, with limits of x and y, and linear forms of N. This table had appeared earlier in *J. d. Math.*, s. 1, v. 16, 1851, p. 273–274, and in *Sochineniia-Oeuvres*, v. 1, 1899, p. 88–89.

R. C. A.

[1] There are now the following six editions of the Theory of Congruences:
(a) *Teoriia Sravnenii* (Theory of Congruences), Diss. St. Petersburg, St. Petersburg, 1849, ix, iii, 281 p.
(b) Second ed., St. Petersburg, 1879, viii, iii, iii, 223, 26 p.
(c) Third ed., St. Petersburg, 1901, xvi, 279 p.
(d) Fourth ed., Moscow-Leningrad, 1944, reviewed above.
(e) *Theorie der Congruenzen*, German transl. by H. Schapira, Berlin, 1889, xviii, 314, 32 p.
(f) *Teoria delle Congruenze*, Italian transl., with additions and a note, by I. Massarini, Rome, 1895, xvi, 295 p.

218[H].—Henry A. Nogrady, *A New Method for the Solution of Cubic Equations*, Ann Arbor, Edwards Bros., 1936, iv, 22, ii, xxx p. 13.5 × 21 cm. Lithoprinted. $1.25.

Given a cubic equation with numerical coefficients,

$$ax^3 + bx^2 + cx + d = 0, \tag{1}$$

the substitution $x = y - b/(3a)$ transforms equation (1) into

$$y^3 + py + q = 0, \tag{2}$$

where $p = (3ac - b^2)/(3a^2)$, $q = 2b^3/(27a^3) - bc/(3a) + d/a$. Applying the transformation $y = qz/p$ to equation (2), we get

$$z^3 + nz + n = 0, \tag{3}$$

where $n = p^3/q^2$. If z_1 is one of the three roots of (3) $n = -z_1^3/(z_1 + 1)$. Hence, while z_1 increases from -1.5 to -1, n decreases from -6.75 to $-\infty$; while z_1 increases from -1 to 0, n decreases from $+\infty$ to 0; while z_1 increases from 0 to 3, n decreases from 0 to -6.75. Thus for $+3 > z_1 > -1.5$, n may take on any value from $+\infty$ to $-\infty$.

In the thirty-page table, for each z_1 (except -1) in this range, at interval .001, the corresponding value of n is given to 6D (except for a few values to a larger number of decimal places). Thus, having reduced a given cubic equation to the form (3), an approximate three-place value of z_1, corresponding to the resulting n, can be read off from the table. Then the other roots of (3), real or imaginary, are found by the formulae

$$z_2 = \tfrac{1}{2}z_1\{-1 + [(z_1 - 3)/(z_1 + 1)]^{\frac{1}{2}}\}, \quad z_3 = \tfrac{1}{2}z_1\{-1 - [(z_1 - 3)/(z_1 + 1)]^{\frac{1}{2}}\}.$$

The author shows that if $z_1 + d$ is the true value of z_1 a six-place value for z_1 may be found by one or more applications of the formula $z_1 + d = (2z_1^3 - n)/(3z_1^2 + n)$.

A tiny 6-page *Pocket Tables for Cubics. A Systematic Method for Algebraic Treatment of Cubic Equations*, 1933, by David Katz, patent attorney, was reviewed in *Scripta Mathematica*, v. 2, 1934, p. 379. The cubic is reduced to the form $z^3 + z = D$.

The numerical solution of cubic equations, with tables, is also set forth by F. Emde,

Tables of Elementary Functions, 1940, p. 38–47, see *MTAC*, p. 384–385. The standard form here is $y^3 + 2 = 3py$. If only one root is real the roots are $y_1 = y' + iy'' = si^{\sigma}$, $y_2 = y' - iy'' = si^{-\sigma}$, $y_3 = -2y' = -2s\cos\sigma$. For $3p = -9.9(.1) + 1(.05)2(.02)2.8(.01)3$, Emde gives y', y'', s, to 4S or 5S, and σ to 4D or 5D. When all the roots are real, the roots y_1, y_2, y_3 are given to 4S or 5S, for $3p = 3(.01)3.2(.02)4(.05)5(.1)10(.5)15$. See also JAHNKE & EMDE, *Tables of Functions . . .*, 1943 and 1945 (*MTAC*, p. 386), Addenda, p. 20–30.

Tables for the solution of the trinomial equations $x^{m+n} \pm ex^m \pm f = 0$, are given in S. GUNDELFINGER, *Tafeln zur Berechnung der reellen Wurzeln sämtlicher trinomischen Gleichungen* ···, Leipzig, 1897.

R. C. A.

219[J, L, M].—I. M. RYZHIK, *Tablitsy Integralov, Summ, Riadov i Proizvedenii* [Tables of Integrals, Sums, Series and Products], Leningrad, OGIZ, 1943. 400 p. 14.6 × 21.5 cm. 25 roubles, paper bound. Edition of 3000 copies.

Since the author felt that no single Soviet or foreign work presented an adequate collection of formulae in integrals, sums, series, and products, for the research mathematician and engineer, the present work with over 5000 formulae was compiled to fill the need. While the work is primarily for such workers, in order to extend the circle of people who might profit from its use, a concise body of indispensable information, supplementing the formulae, is given near the end of the work (p. 339f).

The main sources for the choice of formulae were, (a) for indefinite and elliptic integrals, W. LÁSKA, *Sammlung von Formeln der reinen und angewandten Mathematik*, Braunschweig, 1888–1894; (b) for definite integrals BIERENS DE HAAN, *Tables d'Intégrales Définies*, Amsterdam, 1858–64, and *Nouvelles Tables d'Intégrales Définies*, Leyden, 1867; (c) for sums, series, and products, E. P. ADAMS, *Smithsonian Mathematical Formulae and Tables of Elliptic Functions*, Washington, 1922.

The author tells us that the classification of the material presented great difficulties. The arrangement of the indefinite integrals is essentially that given by G. H. Hardy in his *Integration of Functions of a Single Variable* (Cambridge, second ed., 1928); the author found it desirable, however, to assemble the material on elliptic integrals and functions according to his own arrangement, in Part II. The classification of definite integrals is, with minor modifications, that of Bierens de Haan.

Great attention was paid to special functions, particularly elliptic, cylindrical, spherical. The book contains many formulae pertaining to these functions. Among others are given also formulae of a function introduced by the author namely (p. 296):

$$s(x, y) = \Gamma(x + y - 1)/\Gamma(x)\Gamma(y) = \lim_{n\to\infty} s(x, n)s(y, n)/s(x + y - 1, n)$$

$$= \prod_{k=0}^{\infty} \frac{(x + k)(y + k)}{(x + y - 1 + k)(1 + k)}.$$

The introduction of this function made it possible to simplify and to generalize a number of formulae.

An outline of the contents is as follows:

Part I: Indefinite Integrals (p. 1–80)

1. General formulae; 2. Fundamental integrals; 3. Rational functions; 4. Irrational functions; 5. Trigonometric functions; 6. Exponential functions; 7. Logarithmic functions; 8. Inverse trigonometric functions; 9. Hyperbolic functions.

Part II: Elliptic Integrals and Functions (p. 81–111)

1. Fundamental formulae; 2. Elliptic integrals; 3. Integrals of elliptic functions; 4. Elliptic integrals and Weierstrass functions.

Part III: Definite Integrals (p. 112–239)

1. General formulae; 2. Integrals of elementary functions; 3. Special functions; 4. Multiple integrals.

Part IV: Sums, Series, Products (p. 240–338)

1. Numerical series and products; 2. Fundamental series and products (elementary functions); 3. Special functions.

The theoretical section on *Applications* (p. 339–387) has the following subheadings: 1. Integration in finite form; 2. Approximate methods of integration; 3. Multiple integrals; 4. Convergence of series and products; 5. Classification of series and products; 6. Transformation of series; 7. Connection between series and products; 8. Methods of summation of power series.

Then follow four numerical tables (p. 388–391) of frequent occurrence: (1) $(2n-1)!!/(2n)!! = s(n+1,\ n+1)/2^{2n}$, $n = 1(1)15$; coefficients in the expansion of $(1+x)^{\frac{1}{2}}$; (2) $(2n-1)!!/[(2n)!!(2n+1)] = s(n+1,\ n+1)/[2^{2n}(2n+1)]$, $n = 1(1)15$; (3) $(2n-1)!!/(2n+2)!! = s(n+2,\ n+2)/2^{2n+2}(2n+1)$, $n = 1(1)14$; (4) $(2n-1)!!/[(2n+2)!!(2n+3)] = s(n+1,\ n+1)/(2^{2n+1}s(2n+2,3)]$, $n = 1(1)14$. All of these numerical values are given as fractions and to 10 decimal places; up to $n = 10$, (1) and (3), in decimal forms, are given in P. Barlow, *New Mathematical Tables*, London, 1814, p. 256; up to $n = 15$ the exact decimal values of (1) are given in J. H. Lambert, *Supplementa Tabularum*, Lisbon, 1798, p. 198; also German ed., 1770, p. 210.

Of the 47 titles in the Bibliography (p. 392–393), no dates are given for 41. Most of the titles are well-known works, and it is of interest that only Russian editions are mentioned for works of Courant, La Vallée-Poussin, Euler, Goursat, Granville, Scarborough, Whittaker & Robinson, and Whittaker & Watson. Gray & Matthews' *Treatise on Bessel Functions*, L. B. W. Jolley's *Summation of Series*, B. O. Peirce's *A Short Table of Integrals*, I. Todhunter's *An Elementary Treatise on Laplace's Functions, Lamé's Functions and Bessel's Functions*, and Watson's *A Treatise on the Theory of Bessel Functions*, are naturally included in the list. The only items unknown to the reviewer were two Russian works:

Nina K. Bary, Theory of Series,
I. M. Ryzhik, Special Functions.

Miss Bary has been a professor of mathematics at the Univ. of Moscow since 1932.

A somewhat detailed index of the formulae fills p. 394–400.

The work is undoubtedly one of considerable value for any mathematician to have at hand.

R. C. A.

220[L].—K. E. Bisshopp, "Lateral bending of symmetrically loaded conical discs," *Quarterly Appl. Math.*, v. 2, Oct. 1944, p. 214–217. 17.6 × 25.4 cm. See RMT **202**.

The calculation of the deflection coefficients and stress coefficients depends upon the hypergeometric functions

$$G_1(x) = G_1(1-x) = F\{\tfrac{1}{2}a,\ \tfrac{1}{2}b;\ 1;\ (1-2x)^2\},$$
$$G_2(x) = -\,G_2(1-x) = (1-2x)F\{\tfrac{1}{2}a+\tfrac{1}{2},\ \tfrac{1}{2}b+\tfrac{1}{2};\ 3/2;\ (1-2x)^2\}.$$

The tables give $G_1(x)$, $G'_1(x)$, $G_2(x)$, $G'_2(x)$, for $x = [0(.01).5;$ 6 or 7S].

There are also tables, with 6 or 7S, for the deflection and stress coefficients which are computed by numerical integration from the functions $G_1(x)$, $G_2(x)$, and two auxiliary functions $H_1(x)$, $H_2(x)$. The H-functions are called "subtracting off" functions and are chosen so that the differences, $G_n(x) - H_n(x)$ are bounded uniformly throughout the interval of existence of $G_n(x)$. Some intelligence is used in the choice of these functions.

H. B.

221[L].—N. Karsmenkov in Herbert Buchholz, "Die konfluente hypergeometrische Funktion mit besonderer Berücksichtigung ihrer Bedeutung für die Integration der Wellengleichung in den Koordinaten eines Rotationsparaboloïdes," *Z. angew. Math. Mech.*, v. 23, 1943, p. 106–108, 117. 21.6 × 27.8 cm.

This is a continuation of a former paper, p. 47–58, in which definitions are given of wave-functions $m_{i\tau}^{(p)}(i\zeta)$, $w_{i\tau}^{(p)}(i\zeta)$ suitable for the treatment of problems connected with a paraboloid of revolution. The relation between $m_{i\tau}^{(p)}(i\zeta)$ and Whittaker's confluent hypergeometric function $M_{k,m}(z)$ is

$$m_{i\tau}^{(p)}(i\zeta) = [\tfrac{1}{2}\pi/(i\zeta)]^{\frac{1}{2}}M_{i\tau,\,\frac{1}{2}p}(i\zeta)$$

The tables on p. 106 give from 4 to 7S for

$$(2/\pi)^{\frac{1}{2}}m_{i\tau}^{(0)}(i\zeta)$$

for $\tau = -3(.5)3$ and $\zeta = 1(1)6$. Those on p. 107 give from 5 to 7S for

$$(2/\pi)^{\frac{1}{2}}\partial m_{i\tau}^{(0)}(i\zeta)/\partial\zeta$$

for the same ranges. On p. 106 there is also a short table of the smallest root τ_1 of the equation

$$m_{i\tau}^{(0)}(i\zeta) = 0$$

to 5D for $\zeta = 0(1)6$ and $\zeta = 4.80966$ when $\tau_1 = \pm 0$. A diagram gives a perspective representation of $M_{i\tau,\,0}(i\zeta)/(i\zeta)^{\frac{1}{2}}$ for $-3 < \tau < 2$, $0 < \zeta < 6$. On p. 107 there is a perspective representation of the function

$$\frac{\partial[M_{i\tau,\,0}(i\zeta)/(i\zeta)^{\frac{1}{2}}]}{\partial\zeta}$$

for $\tau = 0$ to 3, $\zeta = 0$ to 6. On p. 108 there is a short table of the first three zeros τ'_1, τ'_2, τ'_3 of $m'^{(0)}_{i\tau}(i\zeta)$. The computations were made by N. KARSMENKOV who also drew the diagrams. Among the asymptotic formulae that are given, mention may be made of one for

$$(i\zeta)^{-\frac{1}{2}(p+1)}M_{i\tau,\,\frac{1}{2}p}(i\zeta)$$

when $\zeta < 0$ and τ is large and positive, a series for $2\sqrt{|\tau_n\zeta|}$ in descending powers of j_{0n}, where $J_0(j_{0n}) = 0$, asymptotic formulae for $M_{\nu,\,\frac{1}{2}p}(z)$ and $W_{\nu,\,\frac{1}{2}p}(z)$ as $z \to \infty$, $\nu \to \infty$ and as $p \to \infty$, some of these being due to Whittaker and Erdélyi. Orthogonal relations are given, one of which is claimed to be new as the arguments of the functions $m_{i\tau_n}^{(p)}(i\zeta)$ are imaginary. It is thought that the series, asymptotic formulae and orthogonal functions given in the paper represent a decisive step forward towards the solution of the practically important problems relating to the paraboloid of revolution. A good bibliography is given (p. 117) but in the light of some information about what is being done in this country the list is far from complete.

H. B.

222[L].—MURLAN S. CORRINGTON & WILLIAM MIEHLE, "Tables of Bessel functions $J_n(x)$ for large arguments," *J. Math. Physics*, M.I.T., v. 24, Feb. 1945, p. 30–50. 17.4 × 25.5 cm.

Five-place tables of $J_n(ms)$ are here given for $n = 0(1)10$, $s = 1(1)20$, and (a) $m = 1(1)10$; (b) $m = \pi(\pi)5\pi$. Also for $m = 1$ and π, $n = 0(1)10$, $s = 21(1)40$. These are the tables to which earlier reference was made in their unpublished form, *MTAC*, p. 285.

The values of $J_n(s)$, $n = 1(1)10$, $s = 1(1)24$ were obtained by rounding off the 18-place values of MEISSEL, published in A. GRAY & G. B. MATHEWS, *A Treatise on Bessel Functions*, London, 1895; second ed. 1922. For higher values of the argument the values were all computed from the asymptotic formula. The auxiliary functions $A_0(x)$, $B_0(x)$, $A_1(x)$, $B_1(x)$, and their tabulations (see *MTAC*, p. 282), were used when possible. Tabular values were computed to 7D or 8D, and later rounded off to 5D. "Where there was any reasonable doubt as to whether to add or drop a half, the function was recalculated more accurately." As a further check comparison was made with the following known tables: (a) H. NAGAOKA, Tokyo, College of Science, *J.*, v. 4, 1891, where there is a table of $J_0(n\pi)$, $n = [1(1)50; 6D]$; a 5-place abridgement is in K. HAYASHI, *Fünfstellige Funktionentafeln* . . ., Berlin, 1930; no errors were found. See *MTAC*, p. 299. (b) Values of $J_n(20)$, $J_n(30)$, $J_n(40)$, $J_n(50)$,

$J_n(100)$, $n = 0, 1$, are given in K. HAYASHI, *Tafeln der Besselschen, Theta-, Kugel-, und anderer Funktionen*, Berlin, 1930; the statement of the paper under review in this connection is incorrect. See *MTAC*, p. 291. (c) L. STEINER, *Math. naturw. Berichte aus Ungarn*, v. 11, 1894, which has a table of $J_1(x)$, $x = [20.1(.1)31(.2)41; 6D]$; on comparing the values for integral argument no discrepancies were found. See *MTAC*, p. 305.

"The mathematical theory of frequency-modulated radio broadcasting shows that the sideband amplitudes of a carrier modulated with sinusoidal variations in frequency are proportional to the Bessel functions of the first kind $J_n(m)$, where n is the sideband number and m is the modulation index. Since m equals the maximum deviation in frequency, D, divided by the audio frequency μ, it is evident that at the lower audio frequencies the ratio D/μ can become quite large. This means that in order to determine the sideband amplitudes it is necessary to use tables of Bessel functions for large values of the argument m."

R. C. A.

223[L].—H. R. F. CARSTEN & Miss N. W. MCKERROW, "The tabulation of some Bessel functions $K_\nu(x)$ and $K_\nu'(x)$ of fractional order," *Phil. Mag.*, s. 7, v. 35, Dec. 1944, p. 816–818. 17.1 × 25.5 cm.

"Under certain conditions, the temperature field within a cylindrical rod, subjected to a sudden change in temperature, may be developed in terms of modified Bessel functions of the second kind, of order $n \pm \frac{1}{4}$, and their derivatives. As values of such functions do not yet appear to have been published, it has been found desirable to prepare tables of these for a range of the variables." Here are tables of $K_{\frac{1}{2}}(x)$, $K_{3/2}(x)$, $K_{5/2}(x)$, $K_{1/4}(x)$, $K_{3/4}(x)$, $-K_{1/4}'(x)$, $-K_{3/4}'(x)$, $x = [.1(.1)5,6(2)10$; mostly 5S]. $K_{\frac{1}{2}}(x) = (\pi/2x)^{\frac{1}{2}}e^{-x} = \lambda$; $K_{3/2}(x) = \lambda[1 + 1/x]$; $K_{5/2}(x) = \lambda[1 + (3/x) + 3/x^2]$. Then $K_{1/4}(x)$, $K_{3/4}(x)$ were computed by Lagrange's interpolation formulae from values of K_0, K_1, $K_2 \cdots$, tabulated in G. N. Watson, *Theory of Bessel Functions*, second ed., 1944, p. 737, and from previously calculated $K_{1/2}(x)$, $K_{3/2}(x) \cdots$. $K_{1/4}'(x) = -1/(4x)K_{1/4}(x) - K_{3/4}(x)$, $K_{3/4}'(x) = -3/(4x)K_{3/4}(x) - K_{1/4}(x)$.

224[L].—K. FRÄNZ & T. VELLAT, "Der Einfluss von Trägern auf das Rauschen hinter Amplitudenbegrenzern und linearen Gleichrichtern," *Elek. Nach. Tech.*, v. 20, 1943, p. 185, 188–189. 21.5 × 28 cm.

The calculations in this paper depend on confluent hypergeometric functions, and a table gives $C_n(x)$, for $n = [1(1)13, 15; 4S]$, with graphs, and $\sum_{n=1}^{15} C_n(x)$, to 3S, and $x = 0$, .1, .15, .25, .4, .6, 1, 1.5, 2.5, 4, 6, 10, where

$$C_n(x) = (1/4\pi) \sum_{k=0(1)n}^{\infty} \frac{x^{2n-4k-2}\Gamma^2(n-k-\frac{1}{2})_1F_1^2(n-k-\frac{1}{2}; n-2k; -x^2)}{(k!)(n-k)![(n-2k-1)!]^2}.$$

H. B.

225[L].—C. TRUESDELL, "On a function which occurs in the theory of the structure of polymers," *Annals Math.*, s. 2, v. 46, Jan. 1945, p. 150. 17.5 × 25.3 cm.

Appell's integral is defined as

$$\phi(x, s) = \frac{x}{\Gamma(s)} \int_0^\infty \frac{t^{s-1}dt}{e^t - x}.$$

The tables are for the functions $\phi(x, -\frac{1}{2})$, $\phi(x, \frac{1}{2})$, $\phi(x, 3/2)$, $x = [.05(.05).95(.01).99, .995(.001)1; 4S]$, and are correct to 4S except in the ranges: for $\phi(x, -\frac{1}{2})$, $.45 \leqq x \leqq .80$; for $\phi(x, \frac{1}{2})$, $.65 \leqq x \leqq .97$; for $\phi(x, 3/2)$, $.75 \leqq x \leqq .999$. In these ranges the tables are correct to within $\pm.001$.

226[L].—[F. VANDREY], Great Britain, Department of Scientific Research and Experiment, Admiralty Computing Service, Tables of Legendre Functions $Q_n(x)$, London, February, 1945, 2 p. text and 2 p. tables. No. SRE/ACS65. 21.5 × 34.3 cm. Mimeographed. This edition of these tables is available only to certain Government agencies and activities.

In *MTAC*, p. 190, appears a review of tables, 1940, computed by F. VANDREY, of the Legendre Function of the second kind, for $n = 0(1)7$, and for $x = [0(.01)1; 5D]$. The present publication is a reprint of these tables with the following corrections:

$Q_2(x)$, $x = .82$, *for* -0.64134, *read* -0.64164
$Q_7(x)$, $x = .51$, *for* -0.31312, *read* -0.31316
$Q_6(x)$, $x = .99$, *for* $+0.03725$, *read* -0.03725
$Q_7(x)$, $x = .99$, *for* -0.07590, *read* -0.21288

"The differences suggest that the last figure given [in the tables] is probably not more than 1 or 2 units in error."

There is a table of $2Q_n(x)$, $n = 1(1)8$, for $x = [0(.01)1; 4D]$ in H. TALLQVIST, *Grunderna af Teorin för Sferiska Funktioner, jämte Användningar inom Fysiken*, Helsingfors, 1905, p. 401–407.

227[L, M].—A. H. A. HOGG, "Equilibrium of a thin slab on an elastic foundation of finite depth," *Phil. Mag.*, s. 7, v. 35, Apr. 1944, p. 270–275. 17 × 25.4 cm.

There are tables, to 4D, for the definite integrals

$$(1/2\pi x)\int_0^\infty J_1(mx)F(m)mdm, \quad b = 0(.1)1,$$

$$(b^2/2\pi x^2)\int_0^\infty J_0(mx)F(m)dm, \quad b = 0(.1)1,2,$$

$$(1/2\pi x^2)\int_0^\infty [mx^2J_1(mx) - m^2x^2J_0(mx)]F(m)dm, \quad b = 0(.1)1,$$

where

$1/F(m) = m^3 + (b/x)^3[(\sinh m \cosh m + mt)/(\sinh^2 m - m^2t^2)]$, $t = 1/(3\text{–}4\,\sigma)$. The values of Poisson's ratio σ used in the tables are .3, .4, .5.

H. B.

228[L, M].—C. P. WELLS & R. D. SPENCE, "The parabolic cylinder functions," *J. Math. Phys.*, M.I.T., v. 24, Feb. 1945, p. 51–64. 17.5 × 25.3 cm.

The tables, p. 60–61, give the functions ${}_eU_a(x)$, ${}_oU_a(x)$ for the ranges $a = -3(1)-1$, $1(1)3$, $x = [0(.1)3; 4D]$. Then follows a table of $\Gamma(\frac{3}{4} + \frac{1}{4}ia)$, $a = 1(1)5$. Graphs are given (p. 62–63) and the first and second roots of ${}_eU_a(x) = 0$ and ${}_oU_a(x) = 0$ are shown in a figure on p. 64.

Formulae for ${}_eU_a(x)$, ${}_oU_a(x)$ are given on p. 51–59. Particular values are

$${}_eU_0(x) = \Gamma(\tfrac{3}{4})(\tfrac{1}{2}x)^{1/2}J_{-1/4}(\tfrac{1}{4}x^2), \quad {}_oU_0(x) = \Gamma(\tfrac{5}{4})(2x)^{\frac{1}{2}}J_{1/4}(\tfrac{1}{4}x^2).$$

The formulae of H. Weber are written in the convenient form

$${}_eU_a(x) = 2^{\frac{1}{2}}c_1\int_0^\infty (\operatorname{sech} u)^{\frac{1}{2}}\cos(\tfrac{1}{2}x^2\tanh u + \tfrac{1}{2}au)du,$$

$${}_oU_a(x) = 2^{-3/2}c_2\int_0^\infty (\operatorname{sech} u)^{3/2}\cos(\tfrac{1}{2}x^2\tanh u + \tfrac{1}{2}au)du,$$

where c_1, c_2 are determined so that ${}_eU_a(0) = 1$, ${}_oU'_a(0) = 1$.

H. B.

229[M].—Georges Goudet & Miss A. M. Gratzmuller, "Divergence par l'effet de la charge d'espace d'un faisceau électronique cylindrique non accéléré," *J. d. Physique et le Radium*, s. 8, v. 5, July 1944, p. 144–147. 21.5 × 28 cm.

$$f(x) = 4\int_0^1 \left[\sinh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1-\sqrt{1-u^2}}}\right) - \sinh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1+\sqrt{1-u^2}}}\right)\right] du,$$

$$\phi_\alpha(x) = E_0/R\rho = \tfrac{1}{2}[f(x) - f(\alpha - x)],\ \psi(\alpha) = \alpha\int_0^{1/2\alpha} \phi_\alpha(u)du.$$

There are tables of $f(x)$, $x = [0(.1)10(1)80;\ 3\text{–}4S]$; $\phi_\alpha(x)$, $\alpha = .5(.5)5,7,10,20$, $x/\alpha = [0(.1).5;\ 3S]$; and of $\psi(\alpha)$, $\alpha = [0(.1)2.5, 2.8(.2)3.2, 3.5, 3.6(.4)4.4, 4.5, 4.8(.2)5.2(.4)\text{-}6.8(.2)7.2, 8(1)20;\ 2\text{–}4S]$. There are graphs of $f(x)$, $0 < x < 2.5$; of $\phi_\alpha(x)$ for the 13 values of α, $0 < x/\alpha < .5$; and of $\psi(\alpha)$.

230[U].—Akademiia͡ Nauk. S.S.S.R., Leningrad, Matematicheskiĭ Institut imeni V. A. Steklova, *Tablitsy dli͡a opredelenii͡a linii polozhenii͡a korabli͡a po radiopelengu.* [Tables for the determination of the line of position of a ship by radio bearings.] Moscow and Leningrad, Academy of Sciences, 1944, 137 p. + errata slip. 12.7 × 20 cm. 10 roubles. 1000 copies in the edition.

These tables permit the rapid determination of lines of position from radio bearings of radio stations within 270 nautical miles of the observer, in latitudes 80° S to 80° N. They were prepared by the Mathematical Institute of the Academy of Sciences, on the request of the Nauchno-ispytatel'nyĭ Hidrograficheskо-shturmanskiĭ Institut [Scientific-experimental Pilot Institute]. The basic ideas are credited to L. A. Li͡usternik, D. A. Vasil'kov and I. I͡a. Akushskiĭ. The computations were carried out by calculating machine at the Institute; it is stated that bearings calculated by the method will be correct to within 15′. Since bearings determined by radio-direction-finding apparatus available for general use before the war were usually given to the nearest degree, and often were in error by several degrees, especially when the sunrise or sunset line lay between the radio station and the observer, the tables would appear to provide ample accuracy.

The book is quite small and convenient to use. Only a single multiplication is required and that could be carried out by slide rule. The paper used in the volume reminds one that it is a war-time product.

In the use of the tables, one starts with an assumed position, P_0, which is chosen in such a way that $\Delta\lambda$, $\Delta\phi$, the differences in longitude and latitude between this position and that of the radio station are integral numbers of degrees. One then seeks in the table the tabular latitude nearest that of the radio station, and with $\Delta\lambda$, $\Delta\phi$ as additional arguments, takes out A_c, the computed bearing; i, a correcting angle; and k, a differential coefficient to be used in allowing for the difference between the observed and computed bearings, $A - A_c$. The line of position can then be laid off in such a way that it makes an angle, $T = A_c + i \pm 90°$ with the local meridian, and that it lies at a perpendicular distance, $k(A - A_c)$ nautical miles from P_0. Rules are given for deciding the side of P_0 on which the line is to be drawn.

The trigonometric relations used in preparing the tables are:

$$\begin{aligned} \cot A_c &= \tan\phi_0 \cos\phi_c \csc\Delta\lambda - \sin\phi_c \cot\Delta\lambda, \\ \tan i &= \sin\phi_c \tan\Delta\lambda, \\ k &= \sin i/(\tan A_c \tan\phi_c), \end{aligned}$$

where λ_0, ϕ_0 are respectively the assumed longitude and latitude of the observer, λ_c, ϕ_c are the longitude and latitude of the radio station and $\Delta\lambda = \lambda_0 - \lambda_c$.

It is interesting to note that the tabulated values of latitude are not evenly spaced, nor are they integral degrees; for example, 0° 00′, 3° 25′, 6° 50′, 10° 15′, 12° 49′, 15° 13′, 17° 16′, $\cdots$, 60° 05′, 60° 30′, 60° 55′, 61° 10′, 61° 35′, $\cdots$, 79° 33′, 79° 40′, 79° 47′, 79° 54′.

The values of $\Delta\lambda$ range from 1° to 28°; and $\Delta\phi$ from − 5° to + 5° for $\phi = 0$, $\Delta\lambda = 1°$; to − 1° to + 1° for $\phi = 79°36'$, $\Delta\lambda = 28°$.

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231[U].—Great Britain, H. M. Nautical Almanac Office, *Astronomical Navigation Tables*, Volume Q, Latitudes N 70°–N 79°, Air Publication 1618. London, H. M. Stationery Office, 1945, iv, 341 p. 16.5 × 24.8 cm. These tables are available only to certain Government agencies and activities.

This is the fifteenth and last volume in the series of no. 1618 which has had restricted circulation in this country by the Hydrographic Office, under the number H. O. 218. We have already reviewed the earlier volumes, *MTAC*, p. 82f, each one covering five degrees of latitude, v. A, 0°–4° (no volume lettered I or 0) to v. P, 65°–69°, the fourteenth. The present volume covering 10° is naturally the largest, and is applicable between latitudes 69° 30′ and 79° 30′ north for specially selected stars, and both north and south for the rest of the volume.

R. C. A.

232[U].—Samuel Herrick, "The air almanac refraction tables," U. S. Naval Inst., *Proc.*, v. 70, Sept. 1944, p. 1140–1141. 17 × 25.5 cm.

In this note Herrick shows how, by graphical representation, the advantages of critical tables can be had in the case of double-entry tables. As illustrations, he chose the tables for total refraction and refraction adjustment as given in the *American Air Almanac*. With height above sea level in feet, and observed altitude in degrees as the ordinate and abscissa respectively, one reads the total refraction (or refraction adjustment) directly from the appropriate graph. Herrick constructed his graphs from data in L. J. Comrie, *Hughes' Tables for Sea and Air Navigation* (see *MTAC*, p. 111) and notes that there is a slight discrepancy between the figures given by Comrie and those presented by the *American Air Almanac*.

C. H. Smiley

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT **216** (N.D.R.C.), **217** (Chebyshev), **222** (Corrington & Miehle), **226** (Vandrey); N **43** (Euler, Legendre, Newman, Powell); QR **18** (Hayashi, Roman).

67. James Burgess, "On the definite integral $(2/\pi^{\frac{1}{2}})\int_0^t e^{-t^2}dt$, with extended tables of values," R. So. Edinburgh, *Trans.*, v. 39, 1898, p. 321. In MTE **62**, *MTAC*, p. 429 there was a reference to the present additional list of errors in Burgess' table.

The test of the values of L was based on the relation $L = t\sqrt{2}\cdot F(t\sqrt{2})$, where $F(x)$ is the function tabulated to 24D in W. F. Sheppard, *The Probability Integral*, T. II (BAASMTC, v. 7, Cambridge, Univ. Press, 1939). My interpolations were all based on 18D of $F(x)$ and its reduced derivatives, while all multiples of $\sqrt{2}$ were carried to 20D. Consequently, the final values of L should be correct to 17D. As an additional check the values of L in the interval $3.0 \leqq t \leqq 5.0$ were differenced repeatedly until 14th differences were reached. This procedure failed to reveal any errors other than those unavoidably committed in curtailing the results. For $t = 5.5$ and 6.0 the values were checked by a second calculation.

Thus it was discovered that the following 13 of the 24 L-entries comprising this table of Burgess are in error, some quite seriously:

t	For	Read
3.0	...927	...925
3.1	...156 224	...148 085
3.2	...583	...586
3.3	...178	...179
3.7	...828 628	...829 207
3.8	...548 822 273	...549 029 082
4.0	...029	...028
4.1	...654 473 280	...659 617 985
4.6	...583	...591
4.7	...719 571 814 619	...751 397 867 062
5.0	...287 316	...315 388
5.5	...386 619	...389 857
6.0	...165	...439

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68. H. W. Holtappel, *Tafels van e^x*, Groningen, 1938. Table I: p. 1–100; T. IIA, IIB, IIC, IID: p. 101–102; T. IIIA, IIIB: p. 103–104; T. IIIC: p. 105–114; T. IIID: p. 115–124; T. IIIE: p. 125–130; T. IIIF, IIIG, IV: p. 131–132. See RMT **214**.

Errors in T. I were discovered in the course of proofreading of Holtappel's values against values given to more decimal places either in our own *Tables of the Exponential Function e^x*, 1939, or in Van Orstrand's memoir, "Tables of the exponential function and of the circular sine and cosine to radian argument," 1921. Whenever discrepancies arose, the values in question were recomputed. Since the values of e^x for the ranges from 5 to 10, and from − 2.5 to − 10, were not recomputed, but checked by our differencing process, there may conceivably be some last-place errors in these ranges which we were unable to detect by our technique.

In testing the remaining 32 pages for error, part of the work was done by differencing Holtappel's values, and subsequently recomputing the values indicated by the differencing to be in error. In T. IIIB and IIID, however, the values were actually recomputed.

Table I

	Argument	For	Read
e^x	1.848	...26048	...25947
	1.849	...28920	...28819
	3.144	...67404	...74038
	6.196	492.78...	490.78...
	6.197	492.27...	491.27...
	6.199	491.25...	492.25...
	6.672	...35155	...35145
	6.685	...076 81756	...067 81756
	7.141	...84199	...84119
	7.373	...58935	...58953
	7.581	...39445	...39945
	7.755	...47695	...47659
	8.302	...17236	...17226
	8.361	...98222	...82216
	8.506	...63744	...63774
	9.465	...24458 64430	...22458 64430
e^{-x}	2.331	...84500	...85000
	3.650	...12288	...11288
	4.158	...64 88043	...63 88043
	4.198	...59980	...55980
	5.397	...10510	...01510
	5.701	...344 26212	...334 26212
	7.460	...56572	...56562
	8.565	...010 06636	...019 06636
	9.659	...38433	...38483

There should be unit increases in last figures for e^x, for arguments: 1.149, 1.188, 3.277, 3.752; e^{-x}, for argument: .956. For the following entries there should be unit decreases for e^x, for arguments: 1.364, 2.137, 2.152, 2.846, 5.360, 6.650; for e^{-x}, for arguments: .764, .819, 1.788, 1.844, 1.943, 2.024.

TABLE IIA

Argument	*For* end figures	*Read*
18	4	6
19	3	9
20	37	54
21	29	75
22	474	598
23	145	482
24	4364	5281

There should be unit increases of last figures for arguments: 12–15, 17; and unit decreases for arguments: −18, −19, −21.

TABLE IIC

There should be unit increases of last figures for arguments: .06, .08; and unit decreases for arguments: .05, .07, −.04.

TABLE IID

For argument: .007, *for* ...68848..., *read* ...66848....

TABLE IIIA

For argument: 1.0, *for* 2.781..., *read* 2.718....

There should be unit decreases of last figures for arguments: 2.3, 2.8, 4.1, 4.6, 7.1, 8.4.

TABLE IIIB

There should be unit increases of last figures for arguments: .056, .058, .086, .094, .098.

TABLE IIIC

Argument	*For*	*Read*
.040	...07441...	...07741...
.464	...3905	...3915
.678	...6	...8
.683	...5	...0

There should be unit increases of last figures for arguments: .006, .262, .284, .369, .375, .395, .423, .447, .481, .483, .489, .764, .769, .775, .783, .915, .989; and unit decreases for arguments: .225, .258, .283, .296, .343, .669, .677, .688, .698, .778.

TABLE IIID

Argument	*For* end figures	*Read*	Argument	*For* end figures	*Read*
5	33	83	41	85	95
18	2	0	42	03	13
19	9	7	43	21	31
30	0	3	44	39	49
31	17	21	45	57	67
32	4	8	46	76	85
33	1	5	47	395	404
34	68	72	48	14	22
35	85	90	49	33	41
36	2	7	229	85	76
37	0	4	731	24	14
38	37	42	732	66	56
39	55	60	733	34	24

There should be unit increases of last figures for arguments: 21, 22, 92, 93, 95–98, 111, 113, 114, 116–118, 139, 152, 172, 178, 203, 211, 213–215, 220–222, 224, 226, 250, 704, 797, 798, 809, 814, 850, 857, 863, 895, 937, 942, 944, 951, 962, 964, 966, 971, 975, 994. There should be unit decreases for arguments: 11, 14–17, 20, 68, 69, 84, 86, 105, 108, 109, 122,

126, 131, 133, 165, 167, 168, 198, 199, 202, 207, 244, 247, 258, 266, 268, 275, 278, 287, 302, 305–309, 311, 318, 324, 328, 333, 334, 338, 339, 349, 353, 354, 358, 363, 365, 374, 378, 397, 399, 403, 407, 424, 434, 435, 437–439, 445, 451, 453, 455, 456, 458, 460, 465–467, 474, 496–498, 506–508, 517, 519, 523, 525, 527, 537, 542, 543, 549, 557, 558, 567, 572, 577, 579, 581, 588, 589, 594, 596, 598, 632, 635, 636, 637, 645–648, 665, 667, 669, 686, 693, 730, 735, 741, 743, 744, 746, 751, 757.

TABLE IIIE

Argument	*For*	*Read*	Argument	*For*	*Read*
7	...02250	...02450	532	...51202	...51203
70	...24500	...45000	753	...50057	...50457
335	...12551	...11251	807	...64459	...62459
346	...85401	...85801	955	...01264	...01265

NYMTP

EDITORIAL NOTE: While last-figure unit errors are of no special importance, Holtappel's table is such a good one, they have been noted here for use in a new edition.

69. NYMTP, *Table of Hahn's function* $S_0(a)$; See *MTAC*, RMT **208**, p. 425.

In our table of this function published in the paper of WHINNERY and JAMIESON, corresponding to the argument $a = .05$, *for* 26.924, *read* 26.239.

NYMTP

UNPUBLISHED MATHEMATICAL TABLES

References have been made to unpublished tables *MTAC*, p. 417 (Bickley), Q **15** (Foster), QR **18** (Roman).

35[A].—ROBERT JAMES PORTER (1882–) *Factor Table for the Eleventh Million.* Two independent mss. for the same million calculated during the years 1916–1933, and 1930–1945, and the property of the author, residing at 266 Pickering Road, Hull, England.

Ms. A. 1916–1933 is in book-form, 267 pp., 8 × 13 inches, each accounting for 3750 numbers, but as the multiples of 2, 3, and 5 are omitted, the actual entries on each page number 1000. The entries are in longhand, in black ink, and are arranged in 40 parallel columns of 25 squares each. The lowest prime factor only is listed, the notation being similar to that used by KULIK, *a* representing 7; *b*, 11; *c*, 13; etc., a bar showing a prime number. About half the entries were made by the stencil method, and the remainder (by an adaptation of the "multiple" method) entered from working-sheets; to obtain the places for a given entry, the column and square were calculated up to, and including, the prime 727, and thereafter the actual number itself.

Ms. B. 1930–1945 is also in book-form, 200 pp., 7 × 7 inches, each accounting for 5040 numbers, but as the multiples of 2, 3, 5, and 7 are omitted, the actual entries on each page number 1152. The entries are in longhand, in black ink, and are arranged in 24 parallel columns of 24 squares, each square accommodating two entries. The lowest prime factor only is listed, and in the same notation as used in *Ms. A*. In the present Ms. the stencil method was not used at all. The entries for 11, 13, 17, 19, were made by direct comparison with D. N. Lehmer, *Factor Table for the First Ten Millions*, the entries for 23 to 223 inclusive by applying to the pages numbered slips showing at their edges the number of column and square needed for each entry, and thereafter by the method used in *Ms. A* for column and square.

The two mss. were purposely made different in form to avoid errors due to similarity of position of the places of entry, and were afterwards cross-checked, each discrepancy investigated, and the mss. brought into agreement. The results, subjected so far to only one check by the author, show that the total number of primes in this million is 61,945.

R. J. PORTER

36[A].—H. S. Uhler, *Exact values of $n!$, $n = 201(1)300$*. A photostat of a typed copy (20 leaves) is in the Library of Brown University.

This calculation is an extension of results in the booklet by this author, 12 Hawthorne Ave., Hamden 14, Conn., reviewed in *MTAC*, p. 312.

37[D].—*Table of $(1/x)$ tan x*, manuscript prepared by, and in possession of, the Westinghouse Electric Corporation, Research Laboratories, East Pittsburgh, Pa.

In some computations on transmission line measurements it was found that a table of $(1/x)$ tan x was necessary and this was computed for the following radian arguments: $x = [0(.0001).1(.001)3.15(.01)6.3(.1)10; 4D]$. For values of the parameter up to $x = 2$, the values of the tangent (to 8S) were taken from the NYMTP volume (1943; see *MTAC*, p. 178f), and for $x > 2$, from K. Hayashi, *Fünfstellige Funktionentafeln*, Berlin, 1930, where $x = [0(.01)10; 5D]$. In this manuscript each value of the argument is followed by the value of tan in the table used, followed by the 4-place value of $(1/x)$ tan x. The only previously published table of this function appears to have been the one in Jahnke & Emde, *Tables of Functions*, $x = [0(.01)3.14; 4\text{–}5S]$.

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Editorial Note: Since Hayashi's table referred to above, is only 5-place, a 4-place table derived from it must be uncertain in the last figure; but furthermore, all of Hayashi's tables are unreliable. If the NYMTP volume for tan x, $x = [2(\cdot 1)10; 10D]$, p. 402–403, had been used, much greater security would have been achieved. Then Hayashi's 10-place table of tan x, *Sieben- und mehrstellige Tafeln der Kreis- und Hyperhelfunktionen* . . ., Berlin, 1926, $x = 2(\cdot 01)6\cdot 3$, p. 128(2)180, could be employed for filling in the remaining gap.

38[A].—J. W. Wrench Jr., $\pi^{\pm n}$.

This table of $\pi^{\pm n}$, $n = 1(1)110$, was calculated by involution of π and $1/\pi$, to 206S at least, and corresponding powers were checked by multiplication to yield a product differing from unity by less than 10^{-206}. Incidentally, the value of π^2 as computed by H. S. Uhler to 262D and my approximation of π^{-1}, correct to 253D, appeared in Nat. Acad. Sci., *Proc.*, v. 24, 1938, p. 29; see *MTAC*, p. 55. Subsequently, I extended the approximation of π^{-1} to 358D.

Upon collation of my results with those given by Peters & Stein, *Anhang*, p. 2, of Peters *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, it was found that their tables of $\pi^{\pm n}$, $n = [1(1)32; 32S$ and $32D$ respectively], are entirely free from error.

It is intended that the present table shall provide the basis for an extensive table of $\pi^n/n!$ to be used in evaluating various transcendental functions corresponding to rational multiples of π in the argument.

J. W. Wrench, Jr.

MECHANICAL AIDS TO COMPUTATION

16[Z].—F. J. Maginniss, "Differential analyzer applications," *General Electric Rev.*, v. 48, May, 1945, p. 54–59.

The paper presents a brief description of problems which have been treated on the differential analyzer at the General Electric Co. References to original publications are given for most of the problems outlined. Although the bibliography of papers describing applications is not complete, it is a useful selection covering a wide field of interest and is reproduced below.

H. P. Kuehni & H. A. Peterson, "A new differential analyzer," *Electrical Engineering*, 1944, p. 221–235, discussion p. 431; A.I.E.E., *Trans.*, v. 63, 1944. See *MTAC*, p. 430f.

V. BUSH, "The differential analyzer: a new machine for solving differential equations," Franklin Institute *J.*, v. 212, 1931, p. 447–488.

IRVEN TRAVIS, "Differential analyzer eliminates brain fag," *Machine Design*, July, 1935, p. 15–18.

HANS KRAFT & CHARLES G. DIBBLE, "Some two-dimensional adiabatic compressible flow patterns," *J. Aeronautical Sci.*, v. 11, 1944, p. 283–298.

F. J. MAGINNISS & N. R. SCHULTZ, "Transient performance of induction motors," A.I.E.E., *Trans.*, v. 63, 1944, p. 641–646, discussion p. 1458.

E. G. KELLER, "An analytical theory of landing-shock effects on an airplane considered as an elastic body," *J. Appl. Mech.*, v. 11, Dec. 1944, p. A219–228; A.S.M.E., *Trans.*, v. 66.

C. CONCORDIA, "Network- and differential-analyzer solution of torsional-oscillation problems involving nonlinear springs," *J. Appl. Mech.*, v. 12, Mar. 1945, p. A43–47; A.S.M.E., *Trans.*, v. 67.

C. CONCORDIA and F. J. MAGINNISS, "Inherent errors in the determination of synchronous-machine reactances by test," *Electrical Engineering*, 1945, p. 288–294, A.I.E.E., *Trans.*, v. 64.

I. H. SUMMERS & J. B. MCCLURE, "Progress in the study of system stability," A.I.E.E., *Trans.*, v. 49, 1930, p. 132–158, discussion p. 159–161.

F. W. SEARS, "A study of electron orbits in the Barkhausen-Kurz effect," Franklin Institute, *J.*, v. 209, 1930, p. 459.

V. BUSH & S. H. CALDWELL, "Thomas-Fermi equation solution by the differential analyzer," *Phys. Rev.*, v. 38, 1931, p. 1898–1902.

D. R. HARTREE, "Approximate wave functions and atomic field for mercury," *Phys. Rev.*, v. 46, 1934, p. 738–743.

D. R. SHOULTS, S. B. CRARY & A. H. LAUDER, "Pull-in characteristics of synchronous motors," *Electrical Engineering*, 1935, p. 1385–1395; A.I.E.E., *Trans.*, v. 54.

F. M. GAGER & J. B. RUSSELL, Jr., "A quantitative study of the dynatron," Inst. Radio Eng., *Proc.*, v. 23, 1935, p. 1536–1566.

M. F. MANNING & J. MILLMAN, "Self-consistent field for tungsten," *Phys. Rev.*, v. 49, 1936, p. 848–853.

J. G. BRAINERD & C. N. WEYGANDT, "Unsymmetrical self-excited oscillations in certain simple nonlinear systems," I.R.E., *Proc.*, v. 24, 1936, p. 914–922.

G. LEMAITRE & M. S. VALLARTA, "On the allowed cone of cosmic radiation," *Phys. Rev.*, v. 50, 1936, p. 493–504.

D. R. HARTREE & J. R. WOMERSLEY, "A method for the numerical or mechanical solution of certain types of partial differential equations," R. So. London, *Proc.*, v. 161A, 1937, p. 353–366.

J. W. BUTLER & C. CONCORDIA, "Analysis of series capacitor application problems," *Electrical Engineering*, 1937, p. 975–988; A.I.E.E., *Trans.*, v. 56.

HANS RODER, "Effect of tuned circuits upon a frequency-modulated signal," I.R.E., *Proc.*, v. 25, 1937, p. 1617–1647.

D. M. MEYERS, D. R. HARTREE, & A. PORTER, "The effect of space-charge on the secondary current in a triode," R. So. London, *Proc.*, v. 158A, 1937, p. 23–37.

D. R. HARTREE, "On an equation occurring in Falkner and Skan's approximate treatment of the equations of the boundary layer," Cambridge Phil. So., *Proc.*, v. 33, 1937, p. 223–238.

EDITH CLARKE, C. N. WEYGANDT, & C. CONCORDIA, "Overvoltages caused by unbalanced short circuits; effect of amortisseur windings," *Electrical Engineering*, 1938, p. 453–466, discussion p. 468; A.I.E.E., *Trans.*, v. 57.

D. R. HARTREE & A. PORTER, "The application of the differential analyzer to transients on a distortionless transmission line," I.E.E., *J.*, v. 83, 1938, p. 648–656.

C. COPPLE, D. R. HARTREE, A. PORTER & H. TYSON, "The evaluation of transient temperature distributions in a dielectric in an alternating field," I.E.E., *J.*, v. 85, 1939, p. 56–66.

C. CONCORDIA, S. B. CRARY & E. E. PARKER, "Effect of prime-mover speed governor characteristics on power system frequency variations and tie line power swings," *Electrical Engineering*, 1941, p. 559–567, discussion p. 734f; A.I.E.E., *Trans.*, v. 60.

C. CONCORDIA, C. N. WEYGANDT, & H. S. SHOTT, "Transient characteristics of current transformers during faults," *Electrical Engineering*, 1942, p. 280–285, discussion p. 469; A.I.E.E., *Trans.*, v. 61.

C. CONCORDIA, H. S. SHOTT & C. N. WEYGANDT, "Control of tie line power swings," *Electrical Engineering*, 1942, p. 306–313, discussion p. 395; A.I.E.E., *Trans.*, v. 61.

J. G. BRAINERD & H. W. EMMONS, "Effect of variable viscosity on boundary layers, with a discussion of drag measurements," *J. Appl. Mech.*, v. 9, 1942, p. A1–6; A.S.M.E., *Trans.*, v. 64.

S. H. C.

NOTES

40. CORRECT, BUT—!.—In H. LEVY & L. ROTH, *Elements of Probability*, Oxford, 1936, p. 80, is the following footnote: "For example, if $n = 10$, the error in replacing $(1 - 1/n)^{-n}$ by e does not affect the sixth decimal place." This is a footnote to the word "large" in the text statement, "If n is sufficiently large, $(1 - 1/n)^{-n}$ is approximately e. . . ."

The correct values of $(.9)^{-10}$ and e, to 9D, are respectively as follows: 2.867 97$\underline{1}$ 991 and 2.718 28$\underline{1}$ 828.

University of Minnesota GEORGE J. STIGLER

EDITORIAL NOTE: It may be noted that LEVY & ROTH evidently did not carry out the necessary computations; perhaps they had in mind the well-known fact that when e is expressed as the infinite sum of reciprocals of successive factorials one needs to take only a few terms in order to obtain a fair approximation to the value of e. Indeed, if one confines one's self to $n = 10$, and carries out computations to 9D, then

$$e \doteq 1 + 1/1! + 1/2! + 1/3! + \cdots + 1/10!,$$

and the sum of the terms is 2.718 281 801. In other words, "if $n = 10$, the error in replacing $\sum_{n=0}^{10} 1/n!$ by e does not affect the sixth [or even the seventh] decimal place."

41. EARLY DECIMAL DIVISION OF THE SEXAGESIMAL DEGREE (see N **29**, p. 400f).—In our previous note on this topic we listed seven or eight editions of *De Thiende*, 1585, by Simon Stevin, including Norton's English translation. We forgot to give a reference to the English edition, 8 or 9, of VERA SANFORD, in *A Source Book in Mathematics*, ed. by D. E. SMITH, New York, 1929, p. 20–34. This was translated from no. 4, Girard's French edition of 1634.

R. C. A.

42. FIRST MORTALITY TABLE (see *MTAC*, p. 402f).—A facsimile of R. So. London, *Phil. Trans.*, 1693, p. 600, including Halley's first mortality table, is printed in *Isis*, v. 23, 1935, p. 16.

G. SARTON

Harvard University

43. FRANCIS WILLIAM NEWMAN (1805–1897).—Newman was a younger brother of J. H. NEWMAN (1801–1890), the English cardinal. He had a brilliant career at Oxford where he obtained a double first in classics and

mathematics in 1826. After serving as professor of classical literature in Manchester New College, "the celebrated Unitarian seminary long established at York and the parent of Manchester College, Oxford," from 1840 until 1846, he was appointed professor of Latin at University College, London, where he remained until 1869. The range of his publications was most extraordinary, for example: *Lectures on Logic* (1838), *Lectures on Political Economy* (1851), an English translation of Homer's Iliad (1856), a volume of Longfellow's Hiawatha, somewhat abridged, translated into Latin (1862), *A Handbook of Modern Arabic* (1866), *Translations of English Poetry into Latin Verse* (1868), *A Dictionary of Modern Arabic* (2 v., 1871), *Libyan Vocabulary* (1882), *Comments on the Text of Aeschylus* (1884), *Anglo-Saxon Abolition of Negro Slavery* (1889), and works in history, theology, morals, politics, in addition to publishing nearly a score of mathematical papers and four mathematical books.

These papers are listed in the Royal Society's *Catalogue of Scientific Papers*, the first in 1836, and the last two, in 1883 and 1889, which contain the following important mathematical tables:

I. "Table of the descending exponential function to twelve or fourteen places of decimals," Cambridge Phil. So., *Trans.*, v. 13, part 3, 1883, p. 145–241.
II. "Table of the exponential function e^x to twelve places of decimals," *idem*, v. 14, part 3, 1889, p. 237–249. This is a table for e^x, $x =$ (a) [.1(.1)3; 16D], (b) [.001(.001)2; 12D]. It was presented to the So. in 1887.

T. I was presented to the So. in 1876 and there is an account of it in Cambridge Phil. Soc., *Proc.*, v. 3, 1876, p. 24. It is a table of e^{-x}, $x =$ (a) [0(.1)37; 18D], (b) [0(.001)15.349; 12D], (c) [15.350(.002)17.298; 14D], (d) [17.3(.005)27.635; 14D]. Concerning (a) C. E. VAN ORSTRAND remarks in his "Tables of the exponential function and of the circular sine and cosine to radian argument," Nat. Acad. Sci., Washington, D. C., *Memoirs*, v. 14, no. 5, 1921, p. 6, "The 18-place table is hardly the equivalent of a 16-place table, as the original computation included only 18 decimals." On p. 11, Van Orstrand draws attention to the following errors in the table:

x	*For*	*Read*
3.5	301 0738 34223 18502	301 9738 34223 18501
26.1	... 22985	... 22895
26.4	... 27424	... 24725
26.9	... 72200	... 77201

The first of Newman's mathematical books was *The Difficulties of Elementary Geometry, especially those which concern the Straight Line, the Plane and the Theory of Parallels*, London, 1841, viii, 143 p. In the introduction is the following statement: "This book consists of extracts from one which was intended to form a continuous system of elementary geometry; but . . . the author . . . has determined on selecting those parts which are either wholly new, or wanting in the common treatises."

The other three mathematical books were published after Newman had become an octogenarian. The first of these appeared in two volumes, paged continuously, *Mathematical Tracts*, Part I, 1888, ii, p. 1–80; Part II, 1889, iv, p. 81–139; both parts published in Cambridge, by Macmillan & Bowes.

The following tables are to be found in the *Tracts:*

P. 55–68.—values of A^{-n}, to 20D, $A = 2(1)60$, and the odd numbers 61 to 77, $n = 1, 2, 3, \cdots$, "continued until A^{-n} is about to vanish. For $A = 2$, and 3, only the odd values of n are used, up to 29, instead of to the necessary respective values $n = 65$ and $n = 41$ in order to reach the limit defined above for A^{-n}.
P. 69–79.—values of A^n to 12D, powers of $A = .02(.01).5$, except .1, $n = 1, 2, 3, \cdots$, continued until A^n is about to vanish.
P. 84–85.—tanh x, $x = [.01(.01)1; 12D]$, corrected by J. C. ADAMS.
P. 109.—$1/x - \operatorname{csch} x$, $1 - \operatorname{sech} x$, $\coth x - 1/x$, $\tanh x$, $x = [.9(-.1).1; 16D]$.
P. 110–113.—csch x, sech x, $x = [1(.1)12.6; 16D]$.
P. 114–116.—$\coth x - 1$, $1 - \tanh x$, $x = [1(.1)9.3; 16D]$.
P. 117–118.—$\operatorname{csch} x - 2e^{-x}$, $2e^{-x} - \operatorname{sech} x$, $x = [1(.1)7.4; 16D]$.
P. 119–120.—$\coth x - 1 - 2e^{-2x}$, $2e^{-2x} - 1 + \tanh x$, $x = [1(.1)6.1; 16D]$.
P. 121–123.—$\sigma(x) = -\ln(1 - e^{-2x})$, $k(x) = \ln(1 + e^{-2x})$, $x = [1(.1)9.1; 16D]$.
P. 124.—ln coth x, $x = [1(.1)6; 16D]$.
P. 126.—"$\phi(\rho) = -\log(1 - c^2 \sin^2 \beta)^{\frac{1}{2}}$ in Legendre's Elliptic scale,"
$\rho = [1(.1)6.4; 16D]$. After this table Newman wrote, "Carefully as I have worked at this table for $\phi(\rho)$ I must confess that I myself distrust it, because I have no check on error and am sadly aware how a tired brain may blunder."
P. 129.—$\ln\coth x + \ln\coth 3x + \ln\coth 5x + \cdots$ and
$D(x) = \ln\coth x - \ln\coth 2x + \ln\coth 3x - \ln\coth 4x + \cdots$, $x = [1(.1)6.3; 16D]$.
P. 131.—$D(2x) + D(4x) + D(6x) + \cdots$, $x = [1(.1)6.1; 16D]$.
P. 132.—$\sigma(2x) + \sigma(4x) + \sigma(6x) + \cdots$, $x = [1(.1)4.6; 16D]$.
P. 133.—$D(2x) - D(4x) + D(6x) - D(8x) + \cdots$, $x = [1(.1)6.3; 16D]$.
P. 134.—$D(x) + D(3x) + D(5x) + \cdots$, $x = [1(.1)6; 16D]$.
P. 135–139.—e^{-x}, $x = [.1(.1)37; 18D]$. Also T. I (a), 1883, above.

Newman's next mathematical work, published when he was 84 years of age, was

Elliptic Integrals, Cambridge, Macmillan & Bowes, 1889. xvi, 200 p. On p. 131 is the same table as on p. 126 of the *Tracts*. On p. 132–133 is a table of F′ taken from A. M. LEGENDRE, *Traité des Fonctions Elliptiques*, v. 2, Paris, 1826, p. 289–290.

During the last five years of his life Newman was totally blind. His last published book appearing just before this period, was

The Higher Trigonometry. Superrationals of Second Order, Cambridge, Macmillan & Bowes, 1892. ii, 117 p.
P. 7.—"The late Professor Jarrett introduced the notation $\underline{n|}$ for $(1\cdot 2\cdot 3\cdot 4\ldots n)$." This was THOMAS JARRETT (1805–1882), professor of Arabic at the University of Cambridge, in a publication of 1830. The notation $n!$ seems to have been introduced by C. KRAMP, in 1808. See F. CAJORI, *A History of Mathematical Notations*, v. 2, Chicago, 1929, p. 69, 72f.
P. 12.—$x \cot x = 1 - 2H_1x^2 - 2H_2x^4 - 2H_3x^6 \cdots - 2H_nx^{2n} - \cdots$; values of H_n, $n = [6(1)16; 16D]$. $H_n = 2^{n-1} B_n/(2n)!$. In this connection Newman refers to "the hideous numbers of Bernoulli."
P. 38.—$S_n = 1 + 2^{-n} + 3^{-n} + \cdots$; Newman reprints Legendre's table of $S_n - 1$, $n = [2(1)35; 16D]$, given both in the above mentioned work of

1826, p. 432, and in his *Exercices de Calcul Intégral*, part 4, Paris, 1814, p. 65. J. W. L. GLAISHER gave a table of S_n, $n = [2(1)107; 32D]$ in *Quart. J. Math.*, v. 45, 1914, p. 148–150; this table is reprinted in H. T. DAVIS, *Tables of the Higher Mathematical Functions*, Bloomington, Indiana, v. 2, 1935, p. 244, 218. Comparison of Glaisher's table with Legendre's showed that in the latter there were errors in last figures; there should be a unit decrease for $n = 5$, and unit increases for $n = 7, 10, 11, 16$. Setting $T_n = 1 + 3^{-n} + 5^{-n} + \cdots = (1 - 2^{-n})S_n$, Newman gave a table of $T_n - 1$, $n = [2(1)23; 16D]$. J. W. L. Glaisher gave a table of T_n, $n = [2(1)67; 32D]$, in *Quart. J. Math.*, v. 45, 1914, p. 151–152; this table is reprinted in H. T. Davis, *idem*, p. 245. Comparison of Glaisher's table with Newman's revealed the following Newman errors: $n = 13$, *for* ...6218, *read* ...4218; $n = 17$, *for* ...8400, *read* ...8395; also 5 other last figure unit errors. EULER gave a table for S_n, $n = [2(1)16; 16D]$ in his *Institutiones Calculi Differentialis*, St. Petersburg, 1755, p. 456–457; also in his *Opera Omnia*, s. 1, v. 10, Leipzig, 1913, p. 349, where errors in the first printing are corrected. Euler gave also a table of T_n, $n = [2(2)44; 23D]$ in his *Introductio in Analysin Infinitorum*, 1748, p. 150–151; also in his *Opera Omnia*, s.1, v. 8, Leipzig, 1922, where errors in the first edition are corrected.

P. 39.—$2^{-n} - 3^{-n} + 4^{-n} - \cdots$, (a) $n = [2(1)34; 16D]$, (b) $n = [2(2)18; 20D]$. J. W. L. Glaisher gave a table of this function $n = [1(1)107; 32D]$ in *Quart. J. Math.*, v. 45, 1914, p. 156–158. Comparison of this table with Newman's revealed the following discrepancies in the latter's: in (a) $n = 3$, *for* ...5932..., *read* ...5732...; $n = 11$, *for* ...324 7126..., *read* ...285 6501...; also 9 end-figure errors of from 1 to 6 units; in (b), $n = 4$, *for* ...764 082..., *read* ...754 082...; $n = 10$, *for* ...83 8435, *read* ...84 3436; also 7 last-figure errors of 1 to 5 units.

P. 45.—$1 - V_n = 3^{-n} - 5^{-n} + 7^{-n} - \cdots$, $n = [1(1)20; 13D]$. After this table Newman states "Having no check on trivial error, contingent on a tired brain, I have to speak diffidently of this little table." PETERS & STEIN give a table of $1 - V_n$, $n = [1(1)53; 32D]$, in PETERS, *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, *Anhang*, p. 94. Comparison of these tables showed that in Newman's table there were 3 errors, namely: 2 end-figure unit errors, and at $n = 4$, *for* ...5488..., *read* ...5448.... Euler gave a table of V_n, $n = [3(2)13; 7D]$, in his *Opuscula Analytica*, St. Petersburg, v. 2, 1785, p. 251. Newman quotes also a table of $(2/n)\ (1 - V_n)$, $n = [1(2)9; 25D]$, $[11(2)17; 24 - 18D]$ by C. GUDERMANN, *Theorie der Potenzial- oder cyklisch-hyperbolischen Functionen*, Berlin, 1833, p. 72; also in *J. f. d. reine u. angew. Math.*, v. 6, 1830, p. 194.

P. 64–65, 103–104.—LnN, N prime, 11 to 97; 13D, with 14 last-figure errors, from 1 to 70 units. Also (p. 103–104) $\phi(x) = x + 3^{-2}x^3 + 5^{-2}x^5 + 7^{-2}x^7 + \cdots$, and $\psi(x) = 2^{-2}x^2 + 4^{-2}x^4 + 6^{-2}x^6 + \cdots$, $x = [.01(.01).5; 12D]$; these tables were checked by J. C. Adams.

Spence's integral is $L(x) = \int_1^x \ln x\, dx/(x - 1)$, and

$$\begin{aligned} L(1 + x) &= \int_1^x \ln(1 + x)dx/x = x - 2^{-2}x^2 + 3^{-2}x^3 - 4^{-2}x^4 + \cdots \\ &= \phi(x) - \psi(x) \\ -L(1 - x) &= x + 2^{-2}x^2 + 3^{-2}x^3 + 4^{-2}x^4 + \cdots = \phi(x) + \psi(x). \end{aligned}$$

There are tables of $L(1 + x)$ and $-L(1 - x)$, $x = [0(.01).5; 12D]$. Spence's integral occurs in an essay by WILLIAM SPENCE (1777–1815); see his *Mathe-*

matical Essays, edited by JOHN F. W. HERSCHEL, with a biographical sketch of the author by John Galt, London, 1819, or in the first edition of *An Essay on the Theory of the Various Orders of Logarithmic Transcendents . . .*, London and Edinburgh, 1809. ALAN FLETCHER has made a study of the tables of Spence's integral by Spence (p. 24), Newman, and by E. O. POWELL (see *MTAC*, p. 189), in *Phil. Mag.*, s. 7, v. 35, Jan. 1944, p. 16–17. Fletcher shows that there are at least "nine gross errors, all due to faulty addition or subtraction by Newman, and not to errors in the basic values of Adams." The study served "to give confidence in the accuracy of Mr. Powell's table." More than one hundred values in this table, or more than a quarter of the whole were examined. The 7-place value needs decreasing by unity at $x = 1.29$ and increasing by unity at 1.42; also three rounding-off errors of a unit in the seventh decimal place were found between .50 and .67. Powell's claim that the seventh decimal place "should not usually be in error by more than one unit," "therefore appears to be amply justified."

P. 85–87.—Clausen's integral, $\mathrm{Cl}\, x = -\int_0^x \ln(2\sin\frac{1}{2}x)dx = \sin x + 2^{-2}\sin 2x + 3^{-2}\sin 3x + \cdots$, tabulated for $x = [1°(1°)180°; 16D]$, is reprinted from Clausen's article in *J. f. d. reine u. angew. Math.*, v. 8, 1832, p. 300.

P. 101–102.—$\tanh x$, $x = [.01(.01)1; 12D]$, corrected by J. C. ADAMS; same table as p. 84–85 of *Tracts*, part II.

*

Newman's character is vividly drawn by Carlyle in his life of Sterling (1851, part III, chap. 1) of whose son Newman was guardian: "an ardently inquiring soul, of fine University and other attainments, of sharp-cutting, restlessly advancing intellect, and the mildest pious enthusiasm." Material concerning Newman and his works may be found in the following sources:

F. HARRISON, *Realities and Ideals, Social, Political, Literary and Artistic*, New York, 1908, p. 371–377.

J. MCCABE, *Biographical Dictionary of Modern Rationalists*, London, 1920.

R. GARNETT, *Encyclopædia Britannica*, eleventh ed., v. 19, 1911; his quotation from Carlyle is highly unreliable. *Allibone's Critical Dict. of English Literature*, Philadelphia, v. 2, 1870 and *Supplement*, v. 2, 1892. More than four columns of the British Museum, *Catalogue of Printed Books*, 1892, are filled with a list of his writings.

I. G. SIEVEKING, *Memoir and Letters of Francis W. Newman*, London, 1909. His mathematical work is not mentioned here. The frontispiece is a reproduction of a daguerreotype of Newman, taken in 1851; he appears also in a reproduction of a sketch of the Newman family by Maria R. Giberne. There are also a reproduction of a photograph taken in middle life, and two views of a bronze bust, by Mrs. Georgina Bainsmith, presented to the University of London in 1907. A reproduction of a photograph of Newman may also be found in *Illustr. London News*, v. 111, 1897, p. 521.

Since the volumes we have been discussing are not often found in public libraries it may be noted that Newman's *Difficulties of Elementary Geometry* is in the British Museum (B.M.), Cornell University, and University of Michigan (U.M.); copies of *Elliptic Integrals* are in Brown Univ. (B.U.), B.M., Boston Public Library (B.P.L.), and the Mittag-Leffler Library at

Djursholm; *Higher Trigonometry* may be seen in B.M., B.P.L., and Columbia Univ.; Part I of *Mathematical Tracts* is in B.U., and U.M., but both parts are in B.P.L., University of California, and the Mittag-Leffler Library. The Mathematical Association, England, has all four of Newman's mathematical works, including both editions of his *Difficulties of Elementary Geometry*, each printed in 1841, but by different printers. The copy of Spence's rare *Mathematical Essays* in B.P.L. was acquired by Nathaniel Bowditch soon after publication, possibly a presentation copy from the editor, and the first Essay, to which we refer, contains many marginal notes in Bowditch's handwriting. Both of the Spence volumes mentioned are in the New York Public Library, and the 1809 *Essay* is in University of California, Berkeley.

R. C. A.

44. TABLE OF $\frac{1}{2}Wx/V$.—A 5-place table of this function is given (*MTAC*, p. 256) in E. B. ROSA & F. W. GROVER, "Formulas and tables for the calculation of mutual and self-inductance," U. S. Bureau of Standards, *Bulletin*, v. 8, no. 1, 1912, table XXII, p. 226–228; $W = \text{ber}\, x \text{bei}' x - \text{bei}\, x \text{ber}' x$, $V = \text{ber}'^2 x + \text{bei}'^2 x$, $x = 0(.1)5(.2)10(.5)15(1)26(2)50(10)100$, with δ^2. A 3-place adaptation of this table, with Δ, is given on p. 162 of H. B. DWIGHT, *Electric Coils and Conductors*, New York, McGraw-Hill, 1945. Dwight tells us that table XXII was calculated by Grover, and that the adaptation was with his permission.

R. C. A.

QUERIES

15. INTEGRAL AND FUNCTIONAL TABLES.—Are there any tables of $\int_0^z e^{-t^2}\, dt$ and of $e^{-z^2}\int_0^z e^{t^2}\, dt$, where z is complex? Or of $\int_0^x e^{-t^2} \sin at\, dt$, and $\int_0^x e^{-t^2} \cos at\, dt$, where x is real?

F. E. WHITE

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EDITORIAL NOTE: In 1930 RONALD M. FOSTER, of the American Telephone and Telegraph Co. (now of the department of mathematics at Polytechnic Institute of Brooklyn), prepared tables of $\text{erfc}z = (2/\pi^{1/2})\int_z^\infty e^{-t^2}dt$, and of $e^{z^2}\,\text{erfc}z$, $z = x + iy$, for $x = 0(1)3$, $y = 0(1)3$, to 5S. From this material he computed a rather large number of values by a simple method of numerical integration along selected rays in the complex plane. These results were then used to draw contour lines, so that the real and the imaginary parts of the error function could be read off in a rough sort of way for a limited range. Charts I and II (50.7 × 50.7 cm.) are of the real and complex parts of erfc$z$, $0 < x < 2$, $0 < y < 2$; Chart III (38 × 38 cm.) is of real and imaginary parts of erfc$z$, $0 < x < 3$, $0 < y < 3$; Chart IV (38 × 38 cm.) is for absolute value and angle of erfc$z$, $0 < x < 3$, $0 < y < 3$. None of this material has been published. A particular case of the first integral of the Query, $z = \frac{1}{2}\pi^{\frac{1}{2}}(1 + i)u$, may be reduced to functions already tabulated, *MTAC*, p. 250, since we then have $\int_0^z e^{-t^2}dt = (\frac{1}{2}\pi i)^{\frac{1}{2}}[C(u) - iS(u)]$.

QUERIES—REPLIES

17. ROOTS OF THE EQUATION TAN $x = cx$ (Q **8**, p. 203; QR **10**, p. 336). —In A. T. MCKAY, "Diffusion for the infinite plane sheet," Phys. So. London, *Proc.*, v. 44, 1932, p. 22–23, there are tables of real roots, x_n, $n = [1(1)4; 4D]$, of this equation for $c = \pm \tan\lambda$, $\lambda° = 0(5°)90°$. In the case of $c = + \tan\lambda$, there are no roots x_1 for $\lambda < 45°$.

R. C. A.

18. TABLES OF TAN^{-1} (m/n) (Q **14**, p. 431).—I have a 9-place manuscript table of $\tan^{-1}(m/n)$, where m and n are integers ranging from 1 to 26 inclusive. However, there may be errors, as the table has not been checked by use. Photostat copies of this manuscript are permitted. Mr. Pollack may also be interested in my paper, "A table of inverse trigonometric functions in radians," *Terrestrial Magnetism*, v. 40, 1935, which contains (p. 311–312) a table of $\tan^{-1} x$, $x = [0(.01)1; 11D]$, and which also gives the first four coefficients of a power series to be used in interpolating; by reducing to decimals the ratio in which he is interested, an ordinary ten-bank computing machine will furnish results to eleven decimals, with some uncertainty in the last figure.

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EDITORIAL NOTE: The smaller interval of argument in the NYMTP, *Table of Arc Tan x*, Washington, 1942, $x = [0(.001)1; 12D]$, may be even more useful for interpolating than the Roman table. There are also tables, of doubtful accuracy, $x = [0(.00001).001; .001(.0001).0999; 20D; 10D]$ in K. HAYASHI, *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen* . . ., Berlin, 1926.

CORRIGENDA ET ADDENDA

P. 26, l. 1–2, *for* trigonometry by H. GELLIBRAND, Gouda, 1633., *read* Gouda, 1633; see RMT **79**.

P. 232, 234(3), 277, 291, 299, 329, *for* HAURVITZ, *read* HAURWITZ.

P. 235^2, 289, *for* FRANZ, *read* FRÄNZ.

P. 252, l. 24–25, *for* We shall not use the notation $\mathrm{ster}_n x + i\, \mathrm{stei}_n x = H_n(xi^{3/2})$, offered by MCLACHLAN & MEYERS., *read* The notation $\mathrm{ster}_n x + i\, \mathrm{stei}_n x = H_n(xi^{3/2})$ was employed by MCLACHLAN & MEYERS.

P. 253, l. 15–18, *read* Polar forms of the functions have been used by some writers, thus the notation of MCLACHLAN 2, $M_n e^{i\theta_n(x)}$, used above, is a slight modification of the notation employed by KENNELLY, LAWS & PIERCE 1. These writers actually use ρ for this particular modulus of a complex quantity but propose the use of the notation M for the modulus in general. The corresponding notation $N_n e^{i\phi_n(x)}$ of MCLACHLAN 2, is adopted by MCLACHLAN & MEYERS.

P. 273, l. 13, *for* 23*c*, *read* 23*c*/15.

P. 293^2, *for* J. G. JAEGER, *read* J. C. JAEGER.

P. 461, RMT, *read* **93** (Lowan, Salzer & Hillman, Bickley & Miller) 53.

P. 462, MTE **3**, *read* (Cunningham, Euler) 26; and **58**, *read* (Lommel, Watson) 366.

P. 478, *add* Thomas, L. H. 453.

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